# Präsenzaufgabe von Theo 4 vom Montag, den 23.5.2011

## 8 Präsenzaufgabe

a) – $[A, BC] = ABC - BCA$

$B[A, C] + [A, B]C = BAC - BCA + ABC - BAC = ABC - BCA$

– $[A, B] + [A, C] = AB - BA + AC - CA$

$[A, B + C] = A(B + C) + (B + C)A = AB - BA + AC - CA$

b) i) $[a, a^\dagger] = \overset{\dagger}{-} a^\dagger a = \frac{m\omega}{2\hbar}((x + i\frac{1}{m\omega}p)(x - i\frac{1}{m\omega}p) - (x - i\frac{1}{m\omega}p)(x + i\frac{1}{m\omega}p))$

$= \frac{m\omega}{2\hbar}(x^2 + i\frac{1}{m\omega}px - ix\frac{1}{m\omega}p + \frac{1}{m^2\omega^2}p^2 - x^2 + i\frac{1}{m\omega}px - ix\frac{1}{\omega m}p - \frac{1}{m^2\omega^2}p^2)$

$= \frac{m\omega}{2\hbar}(2i\frac{1}{m\omega}px - 2xi\frac{1}{m\omega}p) = \frac{i}{\hbar}(px - xp) = \frac{i}{\hbar}[p, x] = \frac{i}{\hbar}(-i\hbar) = -i^2 = 1$

$\Rightarrow [a, (a^\dagger)^2] = 2a^\dagger$

ii) $[a^2, a^\dagger] = -[a^\dagger, a^2] = -a[a^\dagger, a] - [a^\dagger, a]a = a[a, a^\dagger] + [a, a^\dagger]a = 2a$

iii) $[N, a] = [a^\dagger a, a] = -[a, a^\dagger a] = -a^\dagger[a, a] - [a, a^\dagger]a = -a$

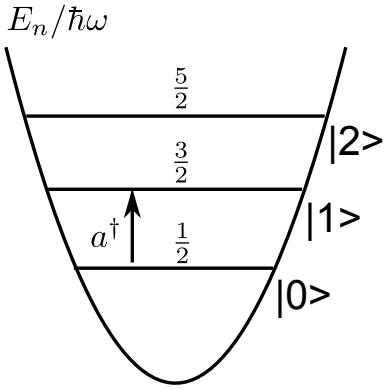
iv) $[N, N^2] = N[N, N] + [N, N]N = 0$

v) $[H, a^\dagger] = -[a^\dagger, \hbar\omega(a^\dagger a + \frac{1}{2})] = -[a^\dagger, \hbar\omega a^\dagger a] - \underbrace{[a^\dagger, \frac{1}{2}\hbar\omega]}_{=0}$

$= -\hbar\omega(a^\dagger[a^\dagger, a] - [a^\dagger, a^\dagger]a) = \hbar\omega a^\dagger$

vi) $[a, f(a^\dagger)] = n(a^\dagger)^{n-1}, \qquad [a^\dagger, f(a)] = 0$

## 9 Präsenzaufgabe




a) $H = \hbar\omega(a^\dagger a + \frac{1}{2})$

b) $H|n> = E_n|n>, \quad N = a^\dagger a, \quad N|n> = n|n>$

$H|n> = \hbar\omega(N + \frac{1}{2})|n> = \hbar\omega(n|n> + \frac{1}{2}|n>) = E_n|n>$

$\Rightarrow E_n = \hbar\omega(n + \frac{1}{2})$

$a^\dagger|n> = \sqrt{n+1}|n+1>, \quad a|n> = \sqrt{n}|n-1>$

c) $x = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger), \quad p = \sqrt{\frac{m\omega\hbar}{2}}(a - a^\dagger)$

$< a_i|\hat{x}|a_j > = \sqrt{\frac{\hbar}{2m\omega}} < a_i|(a + a^\dagger)|a_j > = \sqrt{\frac{\hbar}{2m\omega}}(< a_i|a|a_j > + < a_i|a^\dagger|a_j >)$

$= \sqrt{\frac{\hbar}{2m\omega}}(\sqrt{j}\underbrace{< a_i|a_{j-1} >}_{\delta_{i,j-1}} + \sqrt{i+1}\underbrace{< a_i|a_{j+1} >}_{\delta_{i,j+1}})$

$$\Rightarrow \hat{x} = \begin{pmatrix} 0 & \sqrt{2} & \ddots \\ \sqrt{1} & \ddots & \ddots \\ \ddots & \ddots & 0 \end{pmatrix} \begin{matrix} \\ \\ \downarrow i \end{matrix}$$

j $\longrightarrow$