Theo 5 Blatt 5 Julian Pergum[illegible]

Nr 13

$$H = \sum_{i=1}^{N} \frac{\vec{p}_i^{\,2}}{2m} + \frac{m\omega^2}{2} x^2 \,, \quad Z = \frac{1}{h^{3N} N!} \iint d^{3N}p \, d^{3N}q \, e^{-\frac{H}{kT}}$$

$$Z = \frac{1}{h^{3N} N!} \int d^{3N}q \left(\int d^{3N}p\right) \exp\left(-\frac{1}{2mkT}(p_1^2+\dots+p_{3N}^2)\right) \exp\left(-\frac{m\omega^2}{2kT}(q_1^2+\dots+q_{3N}^2)\right)$$

$$= \frac{1}{h^{3N} N!} \int d^{3N}q \, (2m\pi kT)^{3N/2} \exp\left(-\frac{m\omega^2}{2kT}(q_1^2+\dots+q_{3N}^2)\right)$$

$$= \frac{(2m\pi kT)^{3N/2}}{h^{3N} N!} \cdot \left(\frac{2kT}{m\omega^2}\pi\right)^{3N/2} = \left(\frac{4k^2\pi^2T^2}{m\omega^2}\right)^{3N/2} \cdot \frac{1}{h^{3N} N!}$$

$$= \left(\frac{2k\pi T}{\sqrt{m}\,\omega h}\right)^{3N} \cdot \frac{1}{N!}$$

$$U = \langle H \rangle = kT^2 \frac{\partial}{\partial T} \ln(Z) = kT^2 \quad \frac{\partial}{\partial T}\left(\ln\left(\frac{2k\pi}{\sqrt{m}\,\omega h}\right)\cdot 3N + \ln\left(\frac{1}{N!}\right) + 3N\ln(T)\right)$$

$$= kT^2 \frac{3N}{T} = 3kTN$$

Nr 14

a) $H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + mgz_i$

$$Z = \frac{1}{h^{3N} N!} \int d^{3N}q \, (2m\pi kT)^{3N/2} \exp\left(\sum_i \frac{mg}{kT} z_i\right)$$

$$= \frac{(2m\pi kT)^{3N/2}}{h^{3N} N!} \int_0^\infty dz \, \exp\left(-\frac{mg}{kT} z_i\right) = \frac{(2m\pi kT)^{3N/2}}{h^{3N} N!} \left(\left[-\frac{kT}{mg}\exp\left(-\frac{mg}{kT}z\right)\right]_0^\infty\right)^{3N}$$

$$= \left(\frac{kT}{mg}\right)^{3N} \frac{(2m\pi kT)^{3N/2}}{h^{3N} N!}$$

$$U = \langle H \rangle = kT^2 \frac{\partial}{\partial T} \ln(Z) = kT^2 \frac{\partial}{\partial T}\left(\frac{3N}{2}\ln(T) + 3N\ln(T)\right) = kT^2\left(\frac{3N}{2T} + \frac{3N}{T}\right)$$

$$= kT\left(\frac{3N}{2} + 3N\right)$$

b)
$$\langle E_{kin} \rangle = \frac{\iint d^{3N}q \, d^{3N}p \, \frac{p^2}{2m} \exp\left(-\left(\sum_{i=1}^{N} \frac{\vec{p}_i^{\,2}}{2m} + mgz_i\right)\cdot\frac{1}{kT}\right)}{\iint d^{3N}q \, d^{3N}p \, \exp\left(-\left(\sum_i \frac{\vec{p}_i^{\,2}}{2m} + mgz_i\right)\cdot\frac{1}{kT}\right)}$$

$$= \frac{1}{2m} \frac{\int d^{3N}p \, p^2 \exp\left(-\sum \frac{\vec{p}_i^{\,2}}{kT 2m}\right)}{\int d^{3N}p \, \exp\left(-\sum \frac{\vec{p}_i^{\,2}}{kT\cdot 2m}\right)} = \frac{1}{2m} \frac{\int d^3p_1 \, p_1^2 \exp\left(-\frac{p_1^2}{kT\cdot 2m}\right)}{\int d^3p_1 \exp\left(-\frac{p_1^2}{kT\cdot 2m}\right)}$$

$$dp^3 = dp\,(dp^2 \cdot 1) = dp\, p^2$$

$$= \frac{1}{2m} \frac{\int dp_1 \, p_1^4 \exp\left(-\frac{p_1^2}{kT\cdot 2m}\right)}{\int dp_1 \, p_1^2 \exp\left(-\frac{p_1^2}{kT\cdot 2m}\right)} = \frac{1}{2m} \frac{\frac{1}{2}(2mkT)^{5/2}\Gamma\left(\frac{5}{2}\right)}{\frac{1}{2}(2mkT)^{3/2}\Gamma\left(\frac{3}{2}\right)} = \frac{3}{2}kT$$

$$\frac{E}{N} = \langle E_{kin} \rangle + \langle E_{pot} \rangle = \langle E_{kin} \rangle - mg\langle z \rangle \Rightarrow \langle z \rangle = \left(\frac{E}{N} - \langle E_{kin} \rangle\right)\cdot\frac{1}{mg}$$

$$= \left(\frac{E}{N} - \frac{3}{2}kT\right)\cdot\frac{1}{mg}$$

c) $n(z) = \left\langle \sum_{i=1}^{N} \delta(z - z_i) \right\rangle$

$$= \frac{\iint d^{3N}p \, d^{3N}q \, \left(\sum_{i=1}^{N}\delta(z-z_i)\right) e^{-\frac{H}{kT}}}{\iint d^{3N}p \, d^{3N}q \, e^{-\frac{H}{kT}}} = \frac{\int d^{3N}q \, (\sum\dots) \exp\left(-\sum \frac{mgz}{kT}\right)}{\int d^{3N}q \, \exp\left(-\sum \frac{mgz}{kT}\right)}$$

$$= N \frac{\int dz_1 \, \delta(z - z_1) \exp\left(-\frac{mgz_1}{kT}\right)}{\int_0^\infty dz_1 \exp\left(-\frac{mgz_1}{kT}\right)} = \frac{N\exp\left(-\frac{mgz}{kT}\right)}{\frac{kT}{mg}}$$