Theo 5 Blatt 2

Nr. 4

$M = \sum_i n_i$: Anzahl d. Energiequanten (ununterscheidbar)

$\Gamma(N,M)$: Zahl der Zustände mit E = Möglichkeiten M Energiequanten auf N Zustände zu verteilen

$= \binom{N+M-1}{M} = \frac{(N+M-1)!}{M!\,(N-1)!}$

$S = k\ln(\Gamma(N,M,E)) = k(\ln((N+M-1)!) - \ln(M!) - \ln((N-1)!))$

große $M, N \Rightarrow$ Stirling : $\ln(N!) = \ln(\sqrt{2\pi N}\, N^N e^{-N})$

$= \ln(\sqrt{2\pi N}) + N\ln(N) - N$

$\Rightarrow S = k\Big(\frac{1}{2}\ln(2\pi(N+M)) + (N+M)\ln(N+M) - (N+M)$

$- \frac{1}{2}\ln(2\pi M) - M\ln(M) + M - \frac{1}{2}\ln(2\pi N)$

$- N\ln(N) + N\Big)$

$= k\Big(\frac{1}{2}\ln(2\pi(N+M)) + (N+M)\ln(N+M) - N\ln(N) - M\ln(M)$

$- \frac{1}{2}\ln(2\pi M) - \frac{1}{2}\ln(2\pi N)\Big)$

$E = \frac{N}{2}\hbar\omega + M\hbar\omega \Rightarrow \sigma := \frac{E}{N\hbar\omega} = \frac{1}{2} + \frac{M}{N}$

$S = k\Big(N(1+\frac{M}{N})(\ln(1+\frac{M}{N}) + \ln(N)) - N\ln(N) - N\frac{M}{N}\cdot$

$(\ln(\frac{M}{N}) + \ln(N))$

$+ \frac{1}{2}\ln(2\pi) + \frac{1}{2}\ln(1+\frac{M}{N}) + \frac{1}{2}\ln(N) - \frac{1}{2}\ln(\frac{M}{N}) - \frac{1}{2}\ln(N)$

$- \frac{1}{2}\ln(N)\Big)$

$= k\Big(N\big((1+\frac{M}{N})\ln(1+\frac{M}{N}) - \frac{M}{N}\ln(\frac{M}{N})\big)$

$-\frac{1}{2}\ln(2\pi) + \frac{1}{2}\ln(1+\frac{M}{N}) - \frac{1}{2}\ln(\frac{M}{N}) - \frac{1}{2}\ln(N)\Big)$

$\frac{d}{dx}(x+a)\ln(x+a) = 1 + \ln(x+a)$

$= k\Big(N\big((\sigma+\frac{1}{2})\ln(\sigma+\frac{1}{2}) - (\sigma-\frac{1}{2})\ln(\sigma-\frac{1}{2})\big)$

$- \frac{1}{2}\ln(2\pi) + \frac{1}{2}\ln(\sigma+\frac{1}{2}) - \frac{1}{2}\ln(\sigma-\frac{1}{2}) - \frac{1}{2}\ln(N)\Big)$

$\Rightarrow \cancel{\frac{dS}{dE} = \frac{d\sigma}{dE}} \frac{dS}{d\sigma} = k\Big(N\big(1 + \ln(\sigma+\frac{1}{2}) - 1 - \ln(\sigma-\frac{1}{2})\big)$

$+ \frac{1}{2(\sigma+\frac{1}{2})} - \frac{1}{2\sigma-1} \cancel{- \frac{1}{2}\ln(N)}\Big)$

$\frac{\sigma+\frac{1}{2}}{\sigma-\frac{1}{2}} = \frac{\frac{E}{N\hbar\omega}+\frac{1}{2}}{\frac{E}{N\hbar\omega}-\frac{1}{2}}$

$= k\Big(N\ln\Big(\frac{\sigma+\frac{1}{2}}{\sigma-\frac{1}{2}}\Big) + \frac{2}{1-4\sigma^2} \cancel{- \frac{1}{2}\ln(N)}\Big)$

$\frac{dS}{dE} = \frac{d\sigma}{dE}\frac{dS}{d\sigma} = \frac{1}{N\hbar\omega}\cdot\Big(kN\ln\Big(\frac{\sigma+\frac{1}{2}}{\sigma-\frac{1}{2}}\Big) + \frac{2}{1-4\sigma^2}\Big)$

$= \frac{k}{\hbar\omega}\ln\Big(\frac{\frac{E}{N\hbar\omega}+\frac{1}{2}}{\frac{E}{N\hbar\omega}-\frac{1}{2}}\Big) + \frac{2}{1-4(\frac{E}{N\hbar\omega})^2}\cdot\frac{1}{\hbar\omega N}$

Nr 5

N Teilchen in V: $H = \sum_{i=1}^{3N} \frac{p_i^2}{2m}$

Phasenr. vol: $\Sigma(E) = \frac{1}{N!} \int_{HCE} d\Gamma$ ($\frac{1}{N!}$ da ununterscheidbar)

$\int d\Gamma = \prod_{i=1}^{3N} \frac{dp_i\, dq_i}{(2\pi\hbar)}$, $\int \prod_{i=1}^{3N} dq_i = V^N \Rightarrow \int d\Gamma = V^N \prod_{i=1}^{3N} \frac{dp_i}{2\pi\hbar}$

$\int_{HCE} = \int_E \Theta(E-H) = \int_E \Theta\left(1-\frac{H}{E}\right)$

$\Rightarrow \int d\Gamma = \underbrace{\frac{V^N}{(2\pi\hbar)^{3N}} \int \prod_{i=1}^{3N} dp_i\, \Theta\left(1-\sum_{i=1}^{3N} \frac{p_i^2}{2mE}\right)}_{\text{Darstellung n-dim Kugelvol!}}$

$\Rightarrow \int \ldots = r^s \frac{\pi^{s/2}}{\Gamma(\frac{s}{2}+1)}$ mit $r = \sqrt{2mE}$ wegen $\Theta\left(1-\frac{r^2}{2mE}\right)$

$= \frac{V^N}{(2\pi\hbar)^{3N}} \sqrt{2mE}^{3N} \pi^{3N/2} \cdot \frac{1}{\Gamma(\frac{3N}{2}+1)}$

$\Rightarrow \Sigma(E) = \frac{V^N}{N! (2\pi\hbar)^{3N}} \cdot \frac{\sqrt{2m\pi E}^{3N}}{\Gamma(\frac{3N}{2}+1)}$

Nr 6

$\frac{\partial}{\partial t} \rho = \int d^f p_0 \int d^f q_0 \left(\frac{\partial \rho_0}{\partial p_0} \underbrace{\frac{\partial p_0}{\partial t}}_{-\frac{\partial H}{\partial q}} + \frac{\partial \rho_0}{\partial q_0} \underbrace{\frac{\partial q_0}{\partial t}}_{\frac{\partial H}{\partial p}} \right) \delta(\vec{q} - \vec{Q})\, \delta(\vec{p} - \vec{P})$

$\int d^f p_0\, \delta(\vec{p}_0 - \vec{P}) \vec{p}_0 = \sum_{i=1}^{f} \vec{P}(i) = \sum_{i=1}^{f} \vec{p}_i$

$\Rightarrow \frac{\partial}{\partial t} \rho = \sum_{i=1}^{f} \left(-\frac{\partial \rho_0}{\partial p_i} \frac{\partial H}{\partial q_i} + \frac{\partial \rho_0}{\partial q_i} \frac{\partial H}{\partial p_i} \right)$

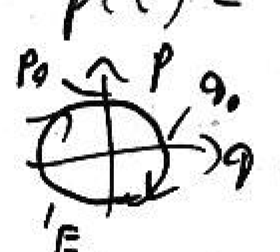
Nr 7

a) $\dot{x} = \frac{\partial H}{\partial p} = \frac{2p}{2m}$, $\dot{p} = \frac{\partial H}{\partial x} = m\omega^2 x$

Angenomen $t=0$ max Ausl $\Rightarrow x(0) = x_{max}, p(0) = 0$

$\Rightarrow x(t) = x_{max} \cos(\omega t)$, $p(t) = -\underbrace{m\omega x}_{p_{max}} \sin(\omega t)$

$E = \frac{p^2}{2m} + \frac{m\omega^2}{2} x^2 \Rightarrow$



$\overline{x^2} = \frac{1}{T} \int_t^{t+T} d\tau\, x(\tau)^2 = \omega \int_t^{t+\frac{2\pi}{\omega}} d\tau\, x_{max}^2 \cos^2(\omega t)$

$= \omega \left[x_{max}^2 \left(\frac{t}{2} + \frac{1}{2\omega} \sin(\omega t) \cos(\omega t) \right) \right]_t^{t+\frac{2\pi}{\omega}}$

$= \omega x_{max}^2 \left(\frac{t}{2} + \frac{1}{2\omega} + \frac{1}{2\omega} \sin(\omega t + 1) \cos(\omega t + 1) \right.$

$\left. - \frac{t}{2} - \frac{1}{2\omega} \sin(\omega t) \cos(\omega t) \right)$

$= \cancel{\omega x_{max}^2 \left(\frac{x}{2} + \frac{1}{4\omega} \sin(2\omega t) \right)} = \omega x_{max}^2 \frac{2 + \sin(2\omega t + 2) - \sin(2\omega t)}{4\omega}$

$T \to \infty \Rightarrow \overline{x^2} = x_{max}^2 (2 + \sin(2) - \sin(0)) \cdot \frac{1}{4}$