12 Besprechung in Analysis 2 zum Blatt 12, zum 7.7.2010

12.1

a) $U = span(v_1, ..., v_k),\ ONB$ von U: $u_1, ..., u_k$

$v_i = \sum_{j=1}^{k} v_{ij} u_j$ Basisdarst. $|(det(v_{ij}))_{i,j=1...k}|\ V = (v_1, ..., v_k),\ O = (u_1, ..., u_k)$.

i-te Spalte: $v_i = \sum_{j=1}^{k} \underbrace{v_{ij}}_{<v_i,u_j>} u_j = \sum_{j=1}^{k} \underbrace{< v_i, u_j >}_{(O^T \cdot V)_{ji}} u_j$=i-te Spalte von $O(O^T V)$. $(V = OO^T V)$

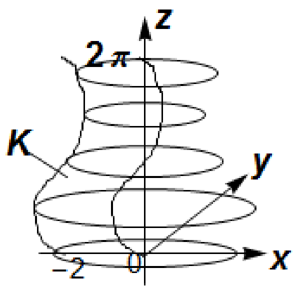
$$O \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_k \end{pmatrix} = x_1 u_1 + ... + x_k u_k$$

b) $|det(v_{ij})| = |det(V^T O)| = \sqrt{det(\underbrace{V^T O}_{k\times k}) \cdot det(\underbrace{O^T V}_{k\times k})}$

mit Det-Mult-Satz: ... $= \sqrt{det(V^T \underbrace{OO^T V}_{V})} = \sqrt{det(V^T V)} = \sqrt{det(g_{ij})},\ \ g_{ij} =< v_i, v_j >$

c) kein Problem (einsetzen), Matrix symmetrisch, determinante ausrechnen $= \sqrt{\frac{111}{2}}$.

12.2



Vol:$\int_K 1 = \int_0^{2\pi} \pi(2 + \sin(z))^2 dz = 9\pi^2$

Masse$= \int_K \varrho = \int_{K\cap\{x\geq 0\}} \varrho + \int_{K\cap\{x<0\}} \varrho = \frac{vol(k)}{2}[1 + \frac{1}{2}] = \frac{3}{4} vol(K) = \frac{27\pi^2}{4}$

$\int_K y = 0 \Rightarrow S_y = 0,\quad \int_K x\varrho(x, y, z) = \int_{K,x\geq 0} x \cdot 1 + \int_{K,x<0} x \cdot \frac{1}{2}$

$= \frac{1}{2} \int_0^{2\pi} (\int_{\substack{k,x\geq 0 \\ x^2+y^2\leq(2-\sin(z))^2}} xd(x, y))dz = \frac{1}{2} \int_0^{2\pi} \int_{-(...)}^{+(...)} (\int_0^{\sqrt{()^2-y}} xdx)dydz = \frac{1}{2} \int_{-(...)}^{+(...)} \frac{1}{2}((...)^2 - y^2)dydz$

$\Rightarrow ... = \frac{1}{3} \int_0^{2\pi} (z + \sin(z))^3 dz = \frac{22}{3}\pi \ \Rightarrow\ S_x = \frac{\frac{22}{3}\pi}{\frac{27\pi^2}{4}} = \frac{88}{81\pi}$

$\int_K z\varrho = \int_0^{2\pi} z \cdot (1 \cdot r(z)^2 \frac{\pi}{2} + \frac{1}{2} r(z)^2 \frac{\pi}{2})dz = \frac{3}{2}\pi \underbrace{\int_0^{2\pi} z \cdot (2 + \sin(z))^2 dz}_{9\pi^2-8\pi\quad (?)} \quad \Rightarrow s_z = \frac{\frac{3\pi}{2}(9\pi^2-8\pi)}{\frac{27\pi^2}{4}}$

$(...)$:=$2 + \sin(z)$

12.3

Koordinatenlinien senkrecht, also Cramer diagonal, Integral harmlos (r=0 bis 1)

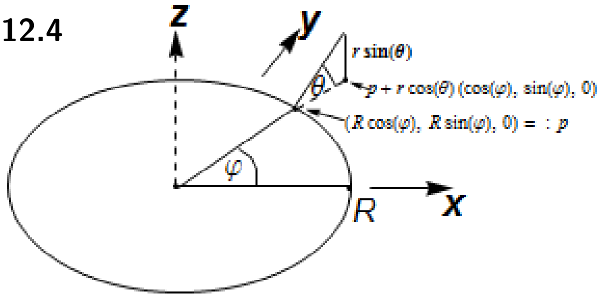
$g_{ij} = \begin{pmatrix} 1 + 4r^2 & 0 \\ 0 & r^2 \end{pmatrix}$, Oberfl.$= \int_0^1 \int_0^{2\pi} \sqrt{1 + 4r^2} d\varphi dr = 2\pi \cdot \frac{2}{3}[(1 + 4r^2)^{\frac{3}{2}}]_0^1 \cdot \frac{1}{8} = \frac{\pi}{6}[5^{\frac{3}{2}} - 1]$

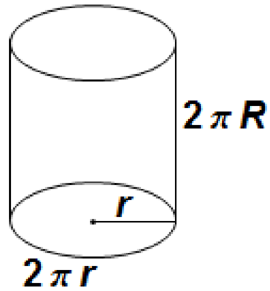
Alternative: $(x, y) \mapsto (x, y, \underbrace{x^2 + y^2}_{h(x)}),\ \nabla h(x, y) = (2x, 2y)$

Vorlesung: $\sqrt{1+||\nabla h||^2}dxdy = \sqrt{1+4x^2+4y^2}$

Fl.$= \int\limits_{x^2+y^2\leq 1} \sqrt{...}dxdy$

12.4





a) $\partial_1\psi(\varphi,\theta) = \begin{pmatrix} -R\sin(\varphi) \\ R\cos(\varphi) \\ 0 \end{pmatrix} + \begin{pmatrix} -r\sin(\varphi)cos(\theta) \\ r\cos(\varphi)\cos(\theta) \\ 0 \end{pmatrix} = (R + r\cos(\theta) \cdot \begin{pmatrix} -\sin(\varphi) \\ \cos(\varphi) \\ 0 \end{pmatrix}$

$\partial_2(\varphi,\theta) = r \begin{pmatrix} -\cos(\varphi)\sin(\theta) \\ -\sin(\varphi)\sin(\theta) \\ \cos(\theta) \end{pmatrix}$

$g_{ij}^{\psi}(\varphi,\theta) =< \partial_i\psi, \partial_j\psi > (\varphi,\theta) = \begin{pmatrix} (R+r\cos(\theta))^2 & 0 \\ 0 & r^2 \end{pmatrix}$

Oberfl.$= r\int 02\pi \int 02\pi (R + r\cos(\theta))d\theta d\varphi = Rr\cdot 4\pi^2 + r^2\cdot 2\pi \underbrace{\int\limits_0^{2\pi} \cos(\theta)d\theta}_{=0}$

12.5

b) $\vec{v} = \nabla U, \quad \int\limits_{\gamma} \vec{v}d\vec{s} = \int\limits_a^b < \underbrace{\vec{v}(\gamma(t))}_{=\nabla U(\gamma(t))}, \dot{\gamma}(t) > dt = \int\limits_0^b \left\{\frac{d}{ds}|_{s=t}[s \mapsto U(\gamma(s))]\right\} dt$

$= U(\gamma(b)) - U(\gamma(a))$ (HDI)