higher hadron physics Ex/Ho 3 Julian Bergmann

~~No~~ 1

a) $x_s = \frac{x_0}{b}$. with x_s as shower length and x_0 as radiation length.

$X_{0,BaF_2} = 2{,}03\,cm$, $X_{0,PbWO_4} = 0{,}89\,cm$

needed b for transmission of the crystal lengths

$L \leq x_s$ & $l_{BaF_2} = 30\,cm$ $\quad l_{PbWO_4} = 20\,cm$

$b \geq \frac{x_0}{l}$: $b_{min,BaF_2} = \frac{2{,}03\,cm}{30\,cm} = 0{,}068$

$b_{min,PbWO_4} = \frac{0{,}89\,cm}{20\,cm} = 0{,}045$

As you can see in PDG p. 267 fig 27,19 b is relatively energy independant and for most materials around 0,5 but never under or near 0,07. This leads to the deduction that there won't be any transmission neither at 500 MeV nor at 1 GeV.

b) In empiric statistics the probability that a particle went through the crystal without collisions is measured by deviding the count of uncollided particles by the total amount of particles that were sent through the crystal.
As photons are traveling with the same speed all the time, the amount of particles is proportional to the intensity of a photobeam.
So, with the formular of intensity loss is

$I(x) = I_0 \exp(-\frac{x}{X_0})$

the probability of a single photon travelling uncollided through the crystal is equal to $\exp(-\frac{L}{X_0})$.

$\Rightarrow P(X_{0,BaF_2} = 2{,}03\,cm, l_{BaF_2} = 30\,cm) = 3{,}8 \cdot 10^{-7}$

$P(X_{0,PbWO_4} = 0{,}89\,cm, l_{PbWO_4} = 20\,cm) = 1{,}74 \cdot 10^{-10}$

c) As the particle shower is caused by the pair production process where the energy of the poton is used to create electrons and positrons, their mass energy must be at least delivered (1,022 MeV). However this would only lead to not moving particles, which is clearly not the case in a shower.
So, using the energy loss formular $\frac{dE}{dx} = -\frac{E}{X_0}$ for charged particles one could assume the kinetic energy of those particles around 1 MeV which would still lead to a total amount of ca. 250 electrons and 250 positrons at 500 MeV beam energy (or 500 at 1 GeV). In a consecutive shower

however, when the produced charged particles merge again and form a new photon, there is much less energy available. So for every process where there is enough energy left after bremsstrahlung and ionization absorbed energy, there ist most likely only one electron-positron pair produced per photon.

d) For position resolution it is important that as much of all electromagnetic showers as possible remain within the crystal. Therfore it is needed to know the lateral shower size. The transverse spread of an electromagnetic cascade is characterized by the Molière radius which is given with good approximation by

$$R_M = \frac{21}{E_c(MeV)} X_0$$

On average only 5% of shower energy transpasses the crystal laterally with a crystal-cylinder of radius $2R_M$. X_0 is the radiation length here, which is given for a specific medium (BaF_2: 2,03cm, $PbWO_4$: 0,89cm).
E_c is the critical energy where the rates of energy loss due to bremsstrahlung and ionization are equal and is given by

$$E_c = \frac{550}{Z} MeV$$

for $Z > 12$ by good approximation.
Concluding this I would suggest a cylinder of

$$2R_{M,BaF_2} = 2 \cdot 4,3cm = 8,6cm$$

$$2R_{M,PbWO_4} = 2 \cdot 2,2cm = 4,4cm$$

radius to use to have 95% of all em cascades remaining within the crystal.

higher hadron physics Ho/Ex 3 Julian Bergmann

N_0 2

a)
$$\left(\langle \frac{dE}{dx} \rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left(\frac{1}{2} \ln \left(\frac{2 m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} \right) - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right) \right.$$
$$T_{max} = \frac{2 m_e c^2 \beta^2 \gamma^2}{1 + 2 m_e \gamma / M + \left(\frac{m_e}{M}\right)^2} = 2 m_e c^2 \beta^2 \gamma^2 \quad \left(\text{for } \frac{2\gamma m_e}{M} \ll 1\right)$$
$$I = (9{,}76\, Z + 58{,}8\, Z^{-0{,}19})\,\text{eV} \quad (\text{for } Z \geq 13)$$
$$\frac{\delta}{2} = \ln\left(\frac{\hbar \omega_p}{I}\right) + \ln(\beta\gamma) - \frac{1}{2}$$
$$\omega_p = \sqrt{\cancel{g} \, g \cdot \langle Z/A \rangle} \cdot 28{,}816\,\text{eV} = \sqrt{4\pi N_e r_e^3}\, \frac{m_e c^2}{\alpha}$$
$$\left. pV = NRT \right)$$

Approximating the gas as fully consitent of argon (as it's already 90%), we can get the stopping power from fig. 302 ~~on~~ p 246 in pdg booklet 2012.
For $\beta\gamma = 3$ the minimum of $-\frac{dE}{dx}$ against $\beta\gamma$ is reached and $-\frac{dE}{dx} \approx 1{,}5 \frac{\text{MeV}}{\text{g} \cdot \text{cm}^2}$ for all three particles.
With this form of $-\frac{dE}{dx}$, you also have to multiply with argon's density of $1{,}662 \cdot 10^{-3} \frac{\text{g}}{\text{cm}^3}$, so the formular of total energy loss becomes
$$\Delta E = -\frac{dE}{dx} \varsigma \Delta x$$
with $\Delta x = 4\,\text{m} - 1\,\text{m} = 3\,\text{m}$, as the angle towards the beam is $90°$.
Therefor ΔE for $\beta\gamma = 3$ is 375 keV.
For $p = 3 \frac{\text{GeV}}{c}$, $\frac{dE}{dx}$ has diffrent values depending on the particle:

particle	$-\frac{dE}{dx}$	ΔE
π	$1{,}8 \frac{\text{MeV cm}^2}{\text{g}}$	450 keV
p	$1{,}5 \frac{\text{MeV cm}^2}{\text{g}}$	375 keV
μ	$1{,}9 \frac{\text{MeV cm}^2}{\text{g}}$	480 keV

b) $\frac{mv^2}{r} \stackrel{!}{=} qvB \iff r = \frac{p}{qB}$

$\beta\gamma = \frac{p}{mc} \Rightarrow p = \beta\gamma m c \Rightarrow$ mass dependend r for $\beta\gamma = 3$

particle	m	r
π	$139{,}6 \frac{\text{MeV}}{c^2}$	2,79 m
p	$938{,}3 \frac{\text{MeV}}{c^2}$	18,78 m
μ	$105{,}7 \frac{\text{MeV}}{c^2}$	2,11 m

$p = 3 \frac{\text{GeV}}{c} \Rightarrow r = 20{,}01\,\text{m}$

c) As shown in a) we can easily differentiate between those particles through energy loss when keeping the same momentum while in b) we can do this with the bending radius in the electromagnetic field by keeping $\beta\gamma$ constant. The bending direction in the magnetic field is also different for particle and anti particle. E.g. p is moved into opposite direction as π^- and same as π^+. π^0 isn't bend in the magnetic field at all. The same goes also for $\mu^{\pm}$.