7 Präsenzblatt vom 29.11.12

7.1 Besprechung Blatt 5

$\mu, \nu \in \{0,1,2,3\} = SK \quad \Rightarrow g_{\mu,\nu} = g^{\mu\nu}$

$\alpha, \beta, \gamma \in \{+,-,1,2\} = LK \quad \Rightarrow \ \tilde{g}^{\alpha\beta};\ \tilde{g}_{\alpha\beta}$

$x^\alpha y_\alpha = x^\mu y_\mu = x^0y^0 - x^1y^1 - x^3y^3 - x^2y^2$

$= x^\alpha \tilde{g}_{\alpha\beta} y^\beta$

$$a^\pm = a^0 \pm a^3 \quad \Leftrightarrow \begin{cases} a^0 = \frac{a^+ + a^-}{2} \\ a^3 = \frac{a^+ - a^-}{2} \end{cases}$$

$\tilde{g}_{\alpha\beta} = (A^T)^{-1} \tilde{g}_{\mu\nu} A^{-1}$

$\Rightarrow x^\alpha \tilde{g}_{\alpha\beta} y^\beta = \frac{1}{2}x^+y^- + \frac{1}{2}x^-y^+ - x^1y^1 - x^2y^2$

$$(\tilde{g}^{\alpha\beta})^{-1} = \tilde{g}_{\alpha\beta} = \left(\begin{array}{c|cccc} & + & - & 1 & 2 \\ \hline + & 0 & 0.5 & 0 & 0 \\ - & 0.6 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 2 & 0 & 0 & 0 & -1 \end{array}\right)$$

$g^{\gamma\alpha} x_\alpha = g^{\gamma\alpha} g_{\alpha\beta} x^\beta = x^\gamma = (g^{\gamma\alpha} g_{\alpha\beta}) x^\beta$

7.2 Präsenzaufgabe 7

a) $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \cdot 1_4$

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\Bigg|_{\mu\nu}$$

$\Rightarrow \{\gamma^\mu, \gamma^\nu\} = 0 \forall \mu \neq \nu \ mit \ \mu, \nu \in \{0,1,2,3\}$

$\gamma^0\gamma^5 = \gamma^0 i\gamma^0\gamma^1\gamma^2\gamma^3 = -i\gamma^0\gamma^1\gamma^0\gamma^2\gamma^3 = -i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^0 = \gamma^5\gamma^0$

$\Rightarrow \{\gamma^\nu, \gamma^5\} = 0$

b) $\gamma^\lambda\gamma_\lambda = \sum_{\alpha\lambda} \gamma^\lambda g_{\lambda\alpha}\gamma^\alpha = g_{\lambda\alpha}\gamma^\lambda\gamma^\alpha = g_{\lambda\alpha}(2g^{\lambda\alpha}1_4 - \gamma^\alpha\gamma^\lambda)$

$= 2g_{\lambda\alpha}g^{\lambda\alpha}1_4 - \gamma^\alpha g_{\alpha\lambda}\gamma^\lambda = 21_4 g_{\lambda\alpha}\gamma^{\lambda\alpha} - \gamma^\alpha\gamma_\alpha$

$2\gamma^\lambda\gamma_\lambda = 2 \cdot 1_4 \sum_\alpha \underbrace{\sum_\lambda g_{\alpha\lambda} g^{\lambda\alpha}}_{g^\alpha_\alpha}$

$\Leftrightarrow \gamma^\lambda\gamma_\lambda = 1_4 \underbrace{\sum_\alpha g^\alpha_\alpha}_{Sp(1_4)} = 4 \cdot 1_4$