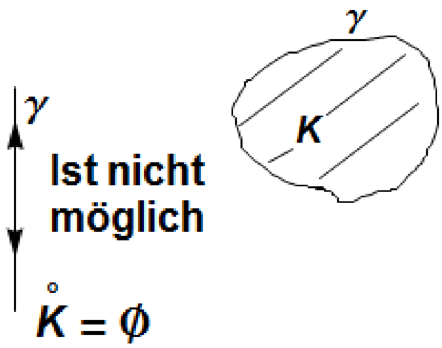
1 Besprechung zu Blatt1 zu Analysis 3

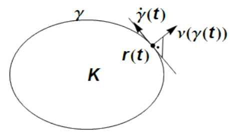
Aufgabe 3

$k = V$
$Kbel : m(t) := \int\limits_K \rho(t,x)dx$
$K \subseteq \mathbb{R}^3$, ∂K-2dim. C^2 Umft. $\dot{m}(t) = -\int\limits_{\partial K}^{\rho} (t,x) < v(t,x), \nu > dx$
$\forall K \subseteq \mathbb{R}^3, \partial K$ 2-dim C^2 Umft gilt:
$-\int\limits_{\partial K} \rho(t,*) < v(t,*), \nu > ds = \dot{m}(t) = \frac{d}{dt}\int\limits_K \rho(t,x)dx = -\int\limits_{\partial K} < \rho(t,*)v(t,*), \nu > ds =$
$-\int\limits_K div(\rho v)(t,x)dx = \int\limits_K \frac{\partial}{\partial t}\rho(t,x)dx$
$\Rightarrow \int\limits_K (\frac{\partial}{\partial t}\rho(t,x) + div(\rho v)(t,x))dx = 0$
$Kbel., \frac{\partial}{\partial t}\rho(t,*) + div(\rho v)(t,*) stetig \Rightarrow \frac{\partial}{\partial t}\rho(t,x) + div(\rho v)(t,x) = 0$

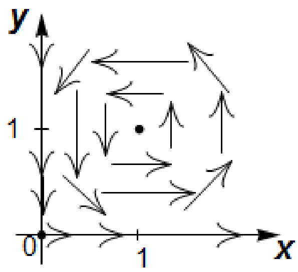
Aufgabe 4

OE. $div(f) > 0$
$\gamma : I \to \mathbb{R}^2$ Lösung von $\dot{x} = f(x)$, periodisch, nicht konstant, dann umschließt das $Bild(\gamma)$
eine kompakte Menge K mit $\overset{\circ}{\neq} \emptyset$
$vol(K) \geq vol(\overset{\circ}{}) > 0$
$\int\limits_K div(f) = \int\limits_{\partial K} < f, \nu > dS$
$\gamma C^1 \Rightarrow \partial K = Bild(\gamma)\ C^1, \exists \nu$
$div(f)$ stetig, K kompakt $\Rightarrow c := \min\limits_{x \in K} div(f(x)) > 0$
$\int\limits_K div(f) \geq \int\limits_K c = c \cdot vol(K) > 0$ (kleiner für max)
$\int\limits_{\partial K} < f, \nu > dS$ Oberflächenintegral der Dim 1, Param: $\gamma : I \to \mathbb{R}^2$
$... = \int\limits_I < f(\gamma(t)), \nu(\gamma(t)) > \sqrt{det(g_{ij})}dt$
$g_{11} = (g_{ij})_{i,j=1} =< \dot{\gamma}(t), \dot{\gamma}(t) >= ||\dot{\gamma}(t)||_2^2$
Beh. $\forall t \in I ist < f(\gamma(t)), \nu(\gamma(t)) >= 0$
$\gamma(t) \in \partial K, T_{\gamma(t)}\partial K = span\{\dot{\gamma}(t)\}$
$\left.\begin{array}{l} \nu(\gamma(t)) in (T_{\gamma(t)}\partial K)^{\perp} = \dot{\gamma}(t)^{\perp} \\ f(\gamma(t)) = \dot{\gamma}(t) \end{array}\right\} \Rightarrow \nu(\gamma(t)) \perp f(\gamma(t)) \Rightarrow < \nu(\gamma(t)), f(\gamma(t)) >= 0$





Aufgabe 5



a) $\alpha, \beta, \gamma, \delta > 0$
$\dot{x} = \alpha x - \beta xy$
$\dot{y} = -\gamma y + \delta xy$
aus der b) erhalten wir: Gleichgewichte $0, (1,1)$ mit
$J_f(0) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $J_f(1,1) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
Taylorpolynom 1. Grades für x, y nahe 0 von
$f(x,y) \approx f(0) + J_f(0)(x,y) = J_f(0)(x,y) = (x, -y)$
$y_0 = 0 \Rightarrow 0$ Lösung von *
$\dot{x} = x \Rightarrow x(t) = ce^t$
$x_0 = 0 \Rightarrow x = 0$
$\dot{y} = -y \Rightarrow y(t) = ce^{-t}$

(x,y) nahe $(1,1)$

$f(x,y) \approx f(1,1) + \underbrace{J_f(1,1)((x,y)-(1,1))}_{Drehmatrix\ mit\ Winkel\ \pi/2} = (-y,x)$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \cos(\pi/2) & -\sin(\pi/2) \\ \sin(\pi/2) & \cos(\pi/2) \end{pmatrix}$$

b) Gleichgewichte: $(x,y) \in (0,\infty)^2$ mit $f(x,y) = 0$

$f : \mathbb{R}^2 \to \mathbb{R}^2, f(x,y) = \begin{pmatrix} \alpha x - \beta xy \\ -\gamma y + \delta xy \end{pmatrix} = \begin{pmatrix} x(\alpha - \beta y) \\ y(-\gamma + \delta x) \end{pmatrix}$

$f(x,y) = 0 \Leftrightarrow (x,y) = 0$ oder $[\alpha - \beta y = 0, -\gamma + \delta x = 0] \Leftrightarrow (x,y) = 0$ oder $(x,y) = (\frac{\gamma}{\delta}, \frac{\alpha}{\beta})$

$J_f(x,y) = \begin{pmatrix} \alpha - \beta y & -\beta x \\ \delta y & -\gamma + \delta x \end{pmatrix}$

$J_f(0) = \begin{pmatrix} \alpha & 0 \\ 0 & \gamma \end{pmatrix}, EW : \alpha, -\gamma$

$J_f(\frac{\gamma}{\delta}, \frac{\alpha}{\beta}) = \begin{pmatrix} 0 & -\frac{\beta\gamma}{\delta} \\ \frac{\delta\alpha}{\beta} & 0 \end{pmatrix} EW : \pm\sqrt{\alpha\gamma}$

charakteristisches Polynom: $\lambda^2 - (\frac{\cancel{\beta}\gamma}{\cancel{\delta}})\frac{\cancel{\delta}\alpha}{\cancel{\beta}} = \lambda^2 + \gamma\alpha$