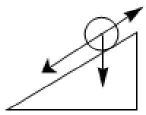
Besprechung zu Blatt 7 zu Analysis 3

Aufgabe 6

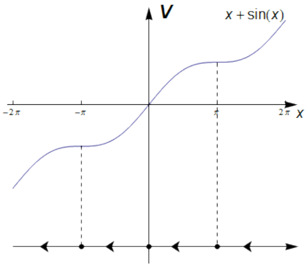


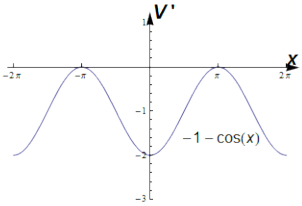
$$\ddot{x} = \underbrace{V'(x(t))}_{Kraft} + \underbrace{\dot{x}(t)}_{Reibung}$$

riesige Kraft, riesige Reibung: $\ddot{x} = \underbrace{\mu}_{\to\infty} \underbrace{(V'(x(t)) + \dot{x}(t))}_{\to 0} \Rightarrow V'(x(t)) + \dot{x}(t) = 0 \Leftrightarrow$

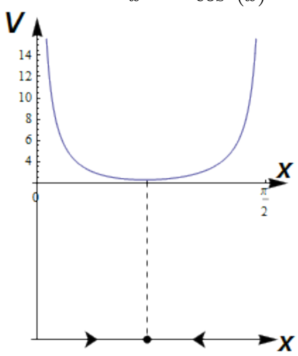
$\dot{x}(t) = -V'(x(t))$

i) $V : \mathbb{R} \to \mathbb{R}, V(x) = x + \sin(x)$
$-V'(x) = -1 - \cos(x)$
Gleichgewichte: $\pi + 2\pi$

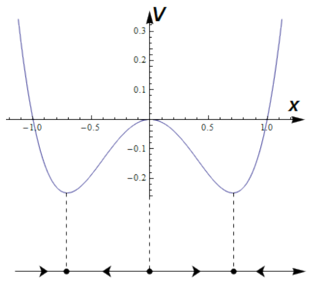




ii) $V(x) = \frac{1}{x} + \tan(x)$
$-V(x) = \frac{1}{x^2} - \frac{1}{\cos^2(x)} = -\frac{x^2 - \cos^2(x)}{x^2 \cos^2(x)}$



iii)

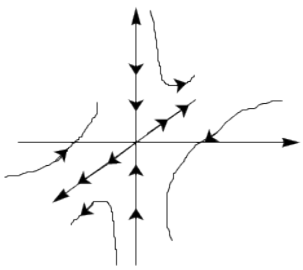


Phasenportraits

Für $A \in \mathbb{R}^{2\times 2}$

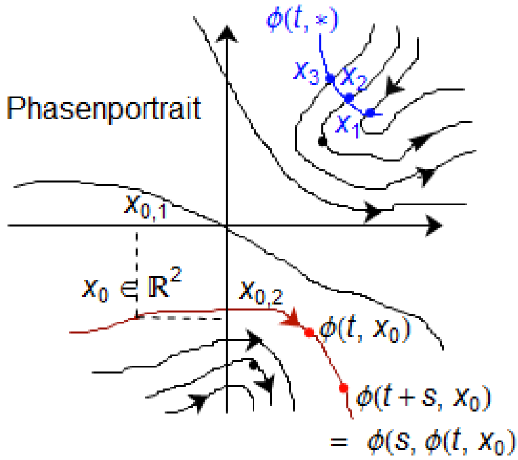
seien 1 EW, $\begin{pmatrix}1\\1\end{pmatrix}$ EV zu 1

-2 EW, $\begin{pmatrix}0\\1\end{pmatrix}$ EV zu -2 $\dot{x} = Ax$



Fluss

$\phi(*, x_0)$



$\dot{\phi}(t,x_0) = f(\phi(t,x_0))$
$\phi(0,x_0) = x_0$
$\phi(s+t,x_0) = \phi(s,\phi(t,x_0))$
x_0^1, x_0^2, x_0^3 nahe beeinander, $f\ C^2 \Rightarrow \phi(t,x_1),\ \phi(t,x_2),\ \phi(t,x_3)$ nahe beiandner $\forall t \in I_{x_0^1}$

Niveaulinien

$E = \frac{\omega^2}{2} + (1-\cos(\varphi))$
Weitere Überlegungen machen aus dem Energieniveau fast das Phasenportrait (eventuell)
Unsere Überlegung mit Pendel: