1 Hausaufgabenbesprechung Blatt 9 23.1.12

1.1 Aufgabe

$X = \{-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\}$, z.z. lsen $f_i \in S_{x,3}$

$f_1(x) = |x|^3 = |x|x^2 = \begin{cases} x^3 & , x \in [0,1] \\ -x^3 & , x \in [-1,0] \end{cases}$

f bel. oft diffbar auf $[-1,1] \setminus \{0\}$

$\lim\limits_{x \to 0, x<0} \frac{f_1(x) - f_1(0)}{x-0} = \lim\limits_{x \to 0, x>0} \frac{f_1(x) - f_1(0)}{x-0}$

$\Rightarrow f_1$ ist diffbar mit Ableitung

$f'(x) = 3 \begin{cases} x^2 & , x \in [0,1] \\ -x^2 & , x \in [-1,0] \end{cases}$

$\lim\limits_{x \to 0} \frac{f_1'(x) - f_1(0)}{x-0} = \lim\limits_{x \to 0, x>0} \frac{f_1'(x) - f_1(0)}{x-0}$

$\Rightarrow f_1'$ ist diffbar mit Ableitung.

$f_1''(x) = \begin{cases} 6x & , x \in [0,1] \\ -6x & , x \in [-1,0] = 6|x| \end{cases} (stetig).$

$\Rightarrow f_1|_{[-1,-1/2]}(x) = -x^3,\ f_1|_{[-1/2,0](x) = -x^3}$

$f_1|_{[0,1/2]}(x) = x^3,\ f_1|_{[1/2,1](x) = x^3}$

$\Rightarrow P_1|_{[x_{i-1}, x_i]}(x) \in \mathbb{P}^3 \forall 1 \leq i \leq 4$

$f_2(x) := (x - \frac{1}{3})_+^3 = \begin{cases} (x - \frac{1}{3})^3 & , x - \frac{1}{3}) \geq 0 \\ 0 & , sonst \end{cases}$

$f_2|_{[0,\frac{1}{2}]}(x) = \begin{cases} (x - \frac{1}{3})^3 & , x \in [\frac{1}{3}, \frac{1}{2}] \\ 0 & , x \in [0, \frac{1}{3}] \end{cases} \notin \mathbb{P}_1^3 \Rightarrow f_2 \notin S_{x,3}$

$f_3(x) = -x + x^3 + 3x^5$

$\Rightarrow f_3|_{[0,\frac{1}{2}]}(x) \notin \mathbb{P}_1^3 \quad \Rightarrow f_3 \notin S_{x,3}$

$f_4(x) = \sum\limits_{n=0}^{3} a_n x^n \ , \quad a_n \in \mathbb{R},\ n = 0, ..., 3$

$f_4 \in C^2([-1,1])$

f_4 ist Polynom vom Grad $\leq 3 \Rightarrow f_4|_{[x_{i-1}, x_i]}(x) \in \mathbb{P}_1^3$

$\Rightarrow f_4 \in S_{x,3}$

$f_5(x) = x^3 e^x \quad \Rightarrow \quad f_5|_{[0,1/2]}(x) \notin \mathbb{P}_1^3$

$\Rightarrow f_5 \notin S_{x,3}$

$f_6(x) = |x|^3 - |x + \frac{1}{3}|^2 = x^2|x| - |x + \frac{1}{3}|^2 \in S_{x,3}$

$f_7(x) = ||x|^3 - |x + \frac{1}{3}|^3|$

$f_7|_{[-1,-1/2]}(x) \notin P_1^3$, da Sprung bei $x = -\frac{1}{3}$ und $\frac{1}{6}$

$f_8(x) = \begin{cases} (x + \frac{1}{2})^2 & , -1 \leq x < -\frac{1}{2} \\ 0 & , -\frac{1}{2} \leq x < \frac{1}{2} \\ -2(x - \frac{1}{2})^3 & , \frac{1}{2} \leq x < 1 \end{cases}$

$f_8|_{[x_{i-1}, x_i]}(x) \in \mathbb{P}_1^3$

$\lim\limits_{\substack{x \to -\frac{1}{2} \\ x < -\frac{1}{2}}} \frac{f_8(x) - f_8(-\frac{1}{2})}{x + \frac{1}{2}} = 0$

GW ex. $f_8|_{[-1,\frac{1}{2}]}$ db. Abl.
$f_8'|_{[-1,-\frac{1}{2}]}(x) = 2(x+\frac{1}{2})$
$\lim\limits_{\substack{x\to-\frac{1}{2}\\ x-\frac{1}{2}}} \frac{f_8'(x)-f_8'(-\frac{1}{2})}{x+\frac{1}{2}} = 2 \neq 0$
$\Rightarrow f_8(x) \notin C^2([-1,1])$

1.2 Aufgabe

$$B_i^0(x) = \begin{cases} \frac{1}{x_{i+1}-x_i} & , x \in [x_i, x_{i+1}] \\ 0 & sonst \end{cases}$$
$B_i^j = \frac{x-x_i}{x_{i+j+1}-x_i} B_i^{j-1}(x) + \frac{x_{i+j+1}-x}{x_{i+j+1}-x_i} B_{i+1}^{j-1}(x)$
$B_0^4(\xi)$, $x_0 = 0.5$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3.1$, $x_4 = 4$, $x_5 = 4.2$
$\xi = 2.6$
$B_0^4(2.6) = \frac{2.6-x_0}{x_5-x_0} B_0^3(2.6) + \frac{x_5-2.6}{x_5-x_0} B_1^3(2.6)$
$B_0^3(2.6) = \frac{2.6-x_0}{x_4-x_0} B_0^2(2.6) + \frac{x_4-2.6}{x_4-x_0} B_1^2(2.6)$
$B_1^3(2.6) = \frac{2.6-x_1}{x_5-x_1} B_1^2(2.6) + \frac{x_5-2.6}{x_5-x_1} B_2^2(2.6)$
$B_0^2(2.6) = \frac{2.6-x_0}{x_3-x_0} B_0^1(2.6) + \frac{x_3-2.6}{x_3-x_0} B_1^1(2.6)$
$B_1^2(2.6) = \frac{2.6-x_1}{x_4-x_1} B_1^1(2.6) + \frac{x_4-2.6}{x_4-x_1} B_1^1(2.6)$
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$B_3^1(2.6) = \frac{2.6-x_3}{x_5-x_3} B_3^0(2.6) + \frac{x_5-2.6}{x_5-x_3} B_4^0(2.6)$
$B_0^0(2.6) = 0$, $B_1^0(2.6)$, $B_2^0(2.6) = \frac{1}{x_3-x_2} = \frac{1}{1.1}$
$B_3^0(2.6) = 0$, $B_4^0(2.6) = 0$, $B_3^1(2.6) = 0$
$B_2^1(2.6) = \frac{3}{11}$, $B_1^1(2.6) = \frac{50}{231}$
$B_0^1(2.6) = 0$, $B_2^2(2.6) = \frac{9}{121}$, $B_1^2(2.6) = \frac{841}{3465}$
$B_0^2(2.6) = \frac{125}{3003}$, $B_1^3(2.6) = \frac{6043}{38115}$
$B_0^3(2.6) \approx= 0.1220601621$, $B_0^4(2.6) \approx 0.1378$

1.3 Aufgabe

Bew. durch vollst. Induktion.
Fr $k = 0$ gilt offensichtlich $h[t_0] = f[t_0]g[t_0]$
Weiter gilt:
$(t_k - t_0)g[t_0...t_k] = h[t_1...t_k] - h[t_0...t_k]$
$= \sum\limits_{i=1}^{k} f[t_1...t_i]g[t_i...t_k] - \sum\limits_{i=0}^{k-1} f[t_1...t_i]g[t_i...t_{k-1}]$
$= \sum\limits_{i=1}^{k} (f[t_1...t_i] - f[t_0...t_{i-1}]g[t_i...t_k]) + \sum\limits_{i=0}^{k-1} f[t_0...t_i](g[t_{i+1}...t_k] - g[t_i...t_{k-1}])$
$= \sum\limits_{i=1}^{k} (t_i - t_0)f[t_0...t_i]g[t_i...t_k]) + \sum\limits_{i=0}^{k-1} (t_k - t_i)f[t_0...t_i]g[t_i...t_k]$
$= \sum\limits_{i=1}^{k} ... + \sum\limits_{i=0}^{k}$

$$== \sum_{i=0}^{k} (t_k - t_0) f[t_0 ... t_i] g[t_i ... t_k])$$