

2 Übungsblatt von Informatik 3 zum Mittwoch, den 4.5.2011

2.1 Aufgabe

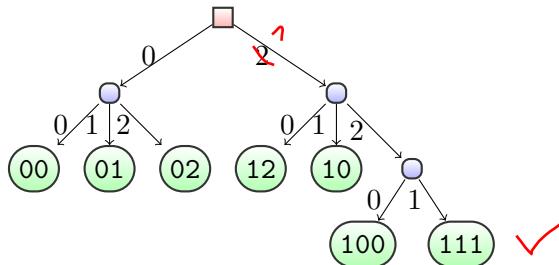
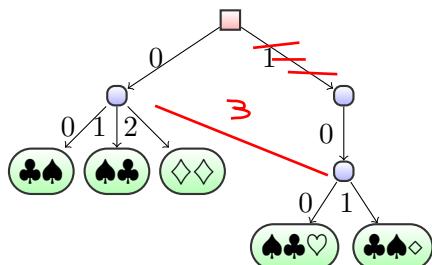
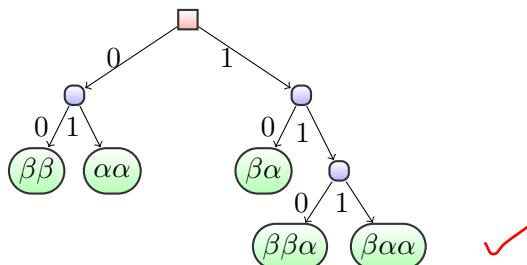
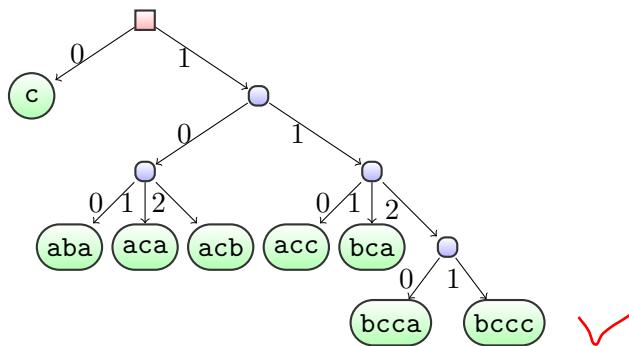
7,5/8

a) $C_1: \sum_i 3^{-n_i} = 5 \cdot 3^{-2} + 2 \cdot 3^{-3} = \frac{17}{27}$ ✓

$C_2: \sum_i 4^{-n_i} = 3 \cdot 4^{-2} + 3 \cdot 4^{-3} = \frac{7}{32}$ ✓

$C_3: \sum_i 2^{-n_i} = 3 \cdot 2^{-2} + 2 \cdot 2^{-3} = 1$ ✓

$C_4: \sum_i 3^{-n_i} = 1 \cdot 3^{-1} + 5 \cdot 3^{-3} + 2 \cdot 3^{-4} = \frac{44}{81}$ ✓

b) $C_1:$  $C_2:$  $C_3:$  $C_4:$ 

2.2 Aufgabe

7,5/8

a) $q = 2, -\frac{\log(p_i)}{\log(q)} < n_i < -\frac{\log(p_i)}{\log(q)} + 1:$

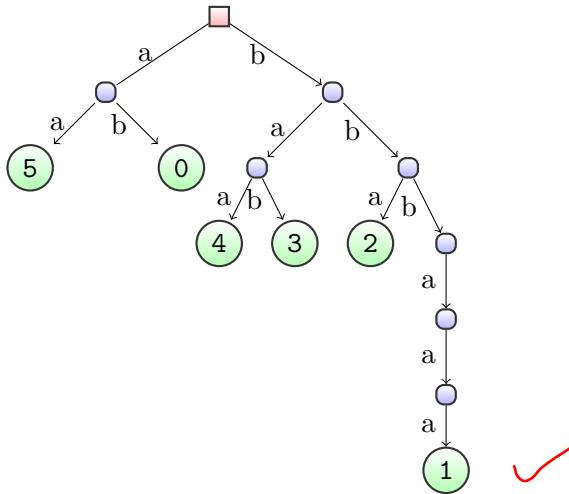
	0	1	2	3	4	5	
p(·)	0.25	0.03	0.14	0.15	0.17	0.26	
	5	0	4	3	2	1	
p(·)	0.26	0.25	0.17	0.15	0.14	0.03	
$-\frac{\log(p)}{\log(q)}$	1.943	2	2.556	2.737	2.837	5.059	
n_i	2	2	3	3	3	6	✓

$$\Rightarrow H(S) = - \sum_{i=1}^6 p_i \log(p_i) = 2.3993 \quad \checkmark$$

$$\Rightarrow \frac{H(S)}{\log(q)} + 1 = \frac{2.3993}{\log(2)} + 1 = 3.3993 \quad \checkmark$$

$$\bar{\lambda}(\phi, S) = 0.26 \cdot 2 + 0.25 \cdot 2 + 0.17 \cdot 3 + 0.15 \cdot 3 + 0.14 \cdot 3 + 0.03 \cdot 6 = 2.58 \quad \checkmark$$

b)



Zunächst könnte man die 1 an die Stelle „bbb“ setzen. Dies ergebe schon

$$\bar{\lambda}(\phi, S) = 2.49 \quad \checkmark$$

(weiterhin decodierbar, da Präfixcode)

Noch näher käme man mit den Verfahren von Shannon oder Hoffmann.

c) a:

$$E(\phi, S) = \frac{H(S)}{\log(q) \cdot \bar{\lambda}(\phi, S)} = \frac{2.3993}{\log(2) \cdot 2.58} = \underline{0.93} \quad \checkmark$$

$$\Rightarrow R(\phi, S) = 1 - E(\phi, S) = 1 - \underline{0.848} = \underline{0.152} \quad \text{? ?}$$

0.07

b:

$$E(\phi, S) = \frac{H(S)}{\log(q) \cdot \bar{\lambda}'(\phi, S)} = \frac{2.3993}{\log(2) \cdot 2.49} = 0.964 \quad \checkmark$$

$$\Rightarrow R(\phi, S) = 1 - E(\phi, S) = 1 - 0.964 = 0.036 \quad \checkmark$$

2.3 Aufgabe

$$\begin{aligned} H(p_1, \dots, p_{m-1}, q_1, q_2) &= H(p_1, \dots, p_m) + p_m \cdot H\left(\frac{q_1}{p_m}, \frac{q_2}{p_m}\right) \\ \Rightarrow H(q_1, q_2) &= H(p_m) + p_m \cdot H\left(\frac{q_1}{p_m}, \frac{q_2}{p_m}\right) \\ \Rightarrow q_1 \log(q_1) + q_2 \log(q_2) &= (q_1 + q_2) \log(q_1 + q_2) + (q_1 + q_2) * \left(\frac{q_1}{q_1 + q_2} \log\left(\frac{q_1}{q_1 + q_2}\right) + \frac{q_2}{q_1 + q_2} \log\left(\frac{q_2}{q_1 + q_2}\right) \right) \\ \Rightarrow q_1 \log(q_1) + q_2 \log(q_2) &= \cancel{(q_1 + q_2) \log(q_1 + q_2)} + q_1 \log(q_1) \cancel{- q_1 \log(q_1 + q_2)} + q_2 \log(q_2) \cancel{- q_2 \log(q_1 + q_2)} \\ \Rightarrow q_1 \log(q_1) + q_2 \log(q_2) &= q_1 \log(q_1) + q_2 \log(q_2) \Rightarrow \checkmark \end{aligned}$$