Exercise 1

1. A 1 GeV/c electron beam hits a proton target. Calculate: *(2 points)*
   (a) The velocity of the CMS
   (b) The momentum of the proton in the CMS
   (c) The available energy

2. Find the names, masses, quark content, dominant decay channel and spectroscopic notation ($^{2S+1}L_J$) for the following N=1 mesons containing only light (u,d,s) quarks: (Hint: Check PDG book(let) for decays; Find all mesons if there is more than one! Remark: C parity (in J^{PC}) is not needed or not defined, thus I removed it from the exercise) *(3 points)*
   (a) $J^P = 1^-$, I=1, positive charge
   (b) $J^P = 0^-$, I=1/2, uncharged
   (c) $J^P = 0^-$, I=0

Homework

1. Explain in your own words an experiment, with which you can measure the form factor of a proton. What beam and target do you use? Where do you place your detector? What exactly do you measure and on which varying variables is it depending on? *(3 points)*

2. Explain in detail and in your own words two methods for measuring the magnetic moment of an hyperon. *(2 points)*

Exercise 1   high hadron physics   Julian Bergmann

No 1

a) $q_e = \left(1\,\text{GeV}, \; \frac{1\,\text{GeV}}{c} - \frac{511\,\text{keV}}{c}, 0, 0\right)$ ✓

$q_p = \left(938\,\frac{\text{MeV}}{c^2}\,c, 0, 0, 0\right)$ ✓

$\vec{p}_{cm} = \frac{\vec{p}_1 + \vec{p}_2}{} = \frac{\vec{p}_e + 1\,\text{GeV}}{}^t = 969\,\frac{\text{MeV}}{c}$ ✓

$0 + 1000 \neq 2$ f $-0{,}25$

$m^* = \sqrt{(q_e + q_p)^2} = 1937\,\frac{\text{MeV}}{c^2}$ ✓

$\beta = \frac{pc^2}{E_m} = \frac{969\,\frac{\text{MeV}}{c}\,c}{1{,}937\,\text{GeV}}$ ff $= 0{,}51$ ✓

b) $p_{p,Lab} = \left(938\,\frac{\text{MeV}}{c}, 0, 0, 0\right)$, $\quad p_{p,cms} = L^\mu{}_\nu$, $\quad L^\mu{}_\nu = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$\gamma = \frac{1}{(1-\beta^2)^{1/2}} = 1{,}163$ ✓

$\Rightarrow p_{p,cms} = \begin{pmatrix} 938\,\text{MeV} \\ 0 \\ 0 \\ 0 \end{pmatrix}^t \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 938\,\text{MeV}\,\gamma \\ -\gamma\beta \cdot 938\,\frac{\text{MeV}}{c} \\ 0 \end{pmatrix}^t$

$= \left(1{,}096\,\text{GeV}, -556\,\frac{\text{MeV}}{c}, 0, 0\right)$ ✓

$\Rightarrow \vec{v}_p = \left(-556\,\frac{\text{MeV}}{c}, 0, 0\right)$ ✓

c) available Energie : in a) calculated : $\sqrt{s} = mc^2 = 1937\,\text{MeV}$ f

$m \neq m_0$ $\quad -0{,}5$

$\frac{1{,}25}{2}$

Exercise 1 — Higher hadron physics — Julian Bergmann

No 2

c) - η, $m = 547{,}8\ \mathrm{MeV}/c^2$, $c_1(u\bar{u}+d\bar{d}) + c_2(s\bar{s})$

$\eta \to 2\gamma$ (39,3 %)

Spectr. not.: $({}^1 0_0)$ ${}^1S_0$

−0,25

- $\eta'(958)$, $m = 957{,}6\ \frac{\mathrm{MeV}}{c^2}$, $c_1(u\bar{u}+d\bar{d}) + c_2(s\bar{s})$

$\eta' \to \pi^+\pi^-\eta$ (44,6 %) Spectr. not.: ${}^1S_0$

- $\eta'(1295)$, $m = 1294\ \mathrm{MeV}$?, ${}^1S_1$

$\eta' \to \eta\pi^+\pi^-$

a) - $\rho^+(1450)$ not $N=1$, $m = 1465\ \mathrm{MeV}$, $u\bar{d}$

−0,5

$\rho \to \pi\pi$ (seen) f, $({}^3 0_1)$ ${}^3S_1$

- $\rho^+(1700)$, $m = 1720\ \mathrm{MeV}$, $u\bar{d}$

$\rho \to 2(\pi^+\pi^-)$, ${}^3D_1$

−0,25

b) - K^0, $m = 497\ \mathrm{MeV}$, $d\bar{s}$

$\bar{K}^0$, ", $\bar{d}s$ ${}^1S_0$

- $K_S^0 \to \pi\pi$ (70 %), $K_L^0 \to \pi^\pm e^\mp \nu$ (40 %)

$2{,}0/3$

Higher hadron physics Exercise 1 Julian Bergmann

No 3

(Deep) elastic

Electrons scattering on protons / hydrogen-Ion (H^+)

?

You are scattering an electronbeam on a thin proton-target with a certain beam energy but different solid angles in which the detector is put. This way you can meassure the cross-section with different $|q|$.

As $\left(\frac{d\sigma}{d\Omega}\right)_{exp} = \left(\frac{d\sigma}{d\Omega}\right)^{*}_{Mott} |F(q^2)|^2$, you can divide the meassure through the calculated Mott cross-section to get the square of abolute value of the proton's form factor.

This shows that the depending variablels are the electron beam's energy and the solid angle of the detector.

You can calculate the $|F(q^2)|^2$-Faktor with the Rosenbluth-Formula

$$\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau \cdot G_M^2(Q^2) \tan^2\left(\frac{\theta}{2}\right)$$

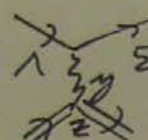
What is a thin proton target for H^+ Ions??

3/3

Higher hadron physics Exercise 1 Julian Bergmann

Nr. 4

The first method is using Primakoff's effect with the dominant decay mode of the Σ^0 hyperon:



with Z as the atomic number of a nucleus.

$\Lambda + Z \to \Sigma^0 + Z$, $\Sigma^0 \to \Lambda^0 + \gamma$

So as Σ^0 is primarily produced with little transverse momentum, measurement of cross-section leads to the determination of its real transition moment.

As the spin precesses around the magnetic field, the "spin precession approach" is another method of determining the magnetic moment of Σ^0.

The precession angle for uncharged hyperons is calculated with $\phi = \frac{2\mu}{\underbrace{\hbar\beta}_{\text{speed of hyperon}}} \underbrace{\int B\,dl}_{\text{field integral}}$, with μ as magnetic moment.

Using the hyperon decay we are able to get the field orientation by observing the process and afterwards, after calculating ϕ with that, get μ.

2/2

Exercises 2.1&2.2 / Homework 2.3&2.4

1. A η meson undergoes a Dalitz decay into $e^+e^-\gamma$. Consider the special case that both the electron and positron get the same kinetic energy. *(2 points)*
   (a) Consider the η beeing at rest and the gamma having a momentum of 200 MeV/c. How large is the opening angle between the electron and position?
   (b) Calculate the invariant mass of the electron-positron pair.

2. Explain (in your own words) Vector Meson Dominance. *(2 points)*
   (a) Where does it play a role?
   (b) What particles are (or can be) involved?
   (c) Sketch a feynmann graph to support your arguments.

3. What are the main differences (in terms of physics) in doing DIS with neutrinos compared to electrons? *(2 points)*

4. In deep-inelastic scattering (DIS) a proton beam of 800 GeV energy collides with an electron beam of 25 GeV energy. You might use reasonable assumptions and simplifications in the following calculations. *(6 points)*
   (a) What CM energy does this correspond to?
   (b) What electron energy would a you need for the same reaction on a fixed proton target?
   (c) What is the "spatial" resolution you have on the proton?
   (d) In what kinematics do you have the maximum Q^2 and how large is it?
   (e) What is measured in Bjorken x?
   (f) What is Bjorken scaling? What do you learn from scaling violations?

higher hadron physiks Exercise 2 Julian Bergmann

No 1

a) $E(e^+) = E(e^-) = \frac{M(\eta) - T(\eta)}{2} = \frac{547{,}8\,MeV - 200\,MeV}{2}$

$= 173{,}9\ MeV$ ✓

ohne e-M

$\Rightarrow q_{cm}(e^-) = (173{,}9\frac{MeV}{c}, 0, 0, 173{,}389\frac{MeV}{c})^T$

$q_{cm}(e^+) = (173{,}9\frac{MeV}{c}, 0, 0, -173{,}389\frac{MeV}{c})^T$

$L = \begin{pmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{pmatrix}$ $\gamma = \frac{E}{M} = \frac{173{,}9\,MeV}{511\,keV} = 340{,}3$

$\gamma = \sqrt{1-\beta^2}^{-1}$ $\Rightarrow \sqrt{1-\frac{1}{\gamma^2}} = \beta$

$\Rightarrow \gamma = 340{,}3,\ \beta = 0{,}999996$

$\Rightarrow q_{lab} = L\, q_{cm}^{(e^-)} = (340{,}3 \cdot 173{,}9\frac{MeV}{c} - 0{,}999996 \cdot 340{,}3 \cdot 173{,}389\,MeV/c,\ 0,\ 0,\ 173{,}389)$

$(\vec{p}_{e^+} + \vec{p}_{e^-})^2 = \vec{p}_{e^+}^{\,2} + \vec{p}_{e^-}^{\,2} + 2\cos(\theta)|p_{e^+}||p_{e^-}|$

$\Rightarrow \frac{E(e^\pm)}{c} = |\vec{p}_{e^+}|$

$(\vec{p}_{e^+} + \vec{p}_{e^-})^2 = (p_{e^+,x} + p_{e^-,x})^2 + (p_{e^+,y} + p_{e^-,y})^2 + (p_{e^+,z} + p_{e^-,z})^2$

Summe = 0 z-Bi li komp. da Koord.sys

$= 40000\,MeV^2/c^2$

$\stackrel{!}{=} (173{,}389\frac{MeV}{c})^2 \cdot 2 + 2\cos(\theta)(173{,}389\frac{MeV}{c})^2$

$\Rightarrow \cos(\theta) = \frac{40000\frac{MeV^2}{c^2} - 2\cdot(173{,}389\frac{MeV}{c})^2}{(173{,}389\frac{MeV}{c})^2}$ ✓

$\Rightarrow \theta = 109{,}56^\circ$ ✓

b) $q_{lab}(e^{+-}) = (173{,}389\frac{MeV}{c}, p_{x,e^\pm}, \pm p_y, 0)$ ✓

$(q_{lab}(e^+) + q_{lab}(e^-))^2 = (2\cdot 173{,}389\frac{MeV}{c})^2 - \underbrace{(p_{x,e^+} + p_{x,e^-})^2}_{=200\frac{MeV}{c}} - \underbrace{(p_y - p_y)^2}_{=0}$

$M^2 = 80254{,}98\,(\frac{MeV}{c^2})^2$

$M =$ 2/2

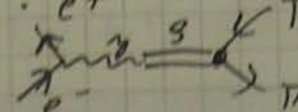
higher hadron physics   Exercise 2   Julian Bergmann

No 2

a) It plays a role for time-like formfactors (e.g. Dalitz decay)
Also important to explain ~~cross-section~~ enhancement of cross-section for $e^+e^- \to \pi^+\pi^-$ (Peak for graph invariant mass against cross-section at ≈ 770 MeV) ✓

this is no explanation! $-\frac{1}{2}$

b) VDM does well for light mesons (π, η, η'), but not for ω. Also it does well for γ into quark-antiquark-pairs. 2 / 0

c)


(intermediate vector mesons) ✓

additional to b)

photon has $J^{PC} = 1^{--}$, so only vector-mesons can be involved. ✓

1,5/2

higher hadron physics Exercise 2 Julian Bergmann

~~Nr~~ 3

2 0

Neutrinos do much less (no) weak ~~and~~ or electromagnetic interaction than electrons. Also the average way of flight without collisions is (also due to the first point) much longer. -1/2

Regarding deep inelastic scattering, electrons can interact with every quark with electromagnetic interaction while neutrinos are using weak interaction (W^{+} transfer) so they are only responsive to d-, $\bar{u}$-, s-, $\bar{c}$-quarks (or counterparts regarding anti-neutrinos).

Because of transversal momentumtransfer ~~...~~ and parityviolation of neutrinos weak interaction ther is an additional partial form factor component, adding up to 3 in total for neutrino-DIP in contrast to elektron-DI~~S~~.

flavor selectivity

1,5/2

Higher hadronphysics Homework 2 Julian Bergmann

~~No~~ 4

a) $q_{el} = \left(25\,\frac{GeV}{c}, 0, 0, \sqrt{\left(25\,\frac{GeV}{c}\right)^2 + \left(511\,\frac{keV}{c}\right)^2}\right) = \left(25\,\frac{GeV}{c}, 0, 0, 25\,\frac{GeV}{c}\right)$

$q_p = \left(800\,\frac{GeV}{c}, 0, 0, \left(\left(800\,\frac{GeV}{c}\right)^2 - \left(938\,\frac{MeV}{c}\right)^2\right)^{1/2}\right) = \left(800\,\frac{GeV}{c}, 0, 0, 800\,\frac{GeV}{c}\right)$

$s = (q_{el}+q_p)^2 c^2 = \left(825\,GeV\right)^2 - \left(775\,GeV\right)^2$

$= 80000\ GeV^2$

b) same reaction $\Leftrightarrow$ same CM-energy

$s = (q_{el}+q_p)^2 c^2 = (E_{el}+E_p)^2 - \left((E_{el}-m_{el}) + \underbrace{(E_p - m_p)}_{=0\ \text{(fixed targ.)}}\right)^2$

$= E_{el}^2 + E_p^2 + 2E_{el}E_p - (E_{el}^2 - 2E_{el}m_{el} + m_{el}^2)$

$= E_p^2 + 2E_{el}E_p + 2E_{el}m_{el} - m_{el}^2$

$\Rightarrow E_{el} = \frac{-E_p^2 + m_{el}^2 + s}{2(E_p + m_{el})}$ fixed proton target : $E_p = m_p$

$\Rightarrow E_{el} = \frac{80000\ GeV^2 + (511\,keV)^2 - (938\,MeV)^2}{2\cdot(938\,MeV + 511\,keV)} = 42620\ GeV = 42{,}6\,TeV$

c) $\Delta x \approx \frac{\hbar}{Q}$, $Q^2 = 2E_{el}E_p(1-\cos(\vartheta))/c^2$

In case of highest Q^2, elastic scattering with $\vartheta = 180°$ with full energy-transfer, the equation $Q^2 = \frac{s}{c^2}$ is valid.

$\Rightarrow \Delta x = \frac{\hbar c}{\sqrt{s}} = 2{,}306\cdot10^{-27}\,s\cdot c = 6{,}914\cdot10^{-19}\,m = 6{,}914\cdot10^{-4}\,fm$

~~(for two beams)~~

~~$\Delta x = \frac{\hbar c}{\sqrt{s}}$~~

4/4

d) If the electron is scattered back at $\vartheta = 180°$ elastically and got the full energy of the proton, Q^2 can be calculated with $Q^2_{max} = \frac{s}{c^2}$.

In this case that means: $Q^2_{max} = 80000\ \frac{GeV^2}{c^2}$

e) The Lorentzinvariant bjorken x is an indicator in deep inelastic scattering, how inelastic the total process of sattering is. For an elastic process x is exactly 1 while otherwise it is between 0 and 1. $(0 < x \leq 1)$

It also indicates that hadrons behave like a collection of point like particles at highenergy scattering. no

f) Bjorken scaling refers to the model where stron interacting particles are behaving like collections of point like particles in high energy scattering.

This leads to the relation that form faktors with the same bjorken-x are independant of Q^2!

The Observation of this leads to the pointlike substruct. of the proton (quarks) skaling violations ?

$$\frac{1{,}25}{2}$$

MP-01, WS 12/13, 20.11.2011

Exercises 3/ Homework 3

See the PDG for properties of crystals, gases and formulas.

1. Gamma detection is done by means of electromagnetic calorimeters. They are build out of different type of scintillating crystals. Two materials which are used are barium-fluoride BaF_2 and lead-tungstate $PbWO_4$. In the following, a crystal length of $30\,cm$ BaF_2 and $20\,cm$ $PbWO_4$ is assumed.

   (a) From the radiation length of the crystals, calculate the fraction of energy a $500\,MeV$ and $1\,GeV$ gamma deposit in the crystal. *(2 points)*

   (b) How large is the propability of the same photons to pass the crystal without starting an electromagnetic shower? *(1 points)*

   (c) How many electron/positrons (order) are created in every shower? *(1 points)*

   (d) What optimal lateral size of the crystals would you suggest for an experiment where position resolution is needed? Give a good reason for your answere. *(1 points)*

2. Most detector types depend on energy loss described by the Bethe-Bloch formula.

   (a) Estimate the total energy loss for charged particles (protons, pions, muons) with $\beta\gamma = 3$ and the same particles with a momentum of 3 GeV/c in the STAR drift chamber, assuming the particles traverse the chamber 90° to the beam direction. Assume further that the bending in the field can be neglected. *(3 points)*

   (b) For the three particles above, calculate the bending radius in the magnetic field (with $\beta\gamma = 3$ and with fixed momentum). *(2 points)*

   (c) What can you conclude from the above result in terms of particle identification? *(1 points)*

   Gas:10%/90% of CH_4/Ar at normal atmosphere; size: outer/inner diameter 4m/1m; magnetic field: 0.5 T

higher hadron physics Ex/Ho 3 Julian Bergmann

~~No.~~ 1

a) $x_s = \frac{x_0}{b}$ with x_s as shower length (mean, max?) and x_0 as radiation length.

$X_{0,BaF_2} = 2{,}03\,cm$, $X_{0,PbWO_4} = 0{,}87\,cm$

needed b for transmission of the crystal lengths

$L \leq x_s$: $L_{BaF_2} = 30\,cm$ $L_{PbWO_4} = 20\,cm$

$b \geq \frac{x_0}{L}$: $b_{min,BaF_2} = \frac{2{,}03\,cm}{30\,cm} = 0{,}068$

$b_{min,PbWO_4} = \frac{0{,}87\,cm}{20\,cm} = 0{,}045$

As you can see in PDG p. 267 fig 27.19 b is relatively energy independant and for most materials around 0,5 but never under or near 0,07. This leads to the deduction that there won't be any transmission neither at 500 MeV nor at 1 GeV.

b) In empiric statistics the probability that a particle went through the crystal without collisions is measured by deviding the count of uncollided particles by the total amount of particles that were sent through the crystal.
As photons are traveling with the same speed all the time, the amount of particles is proportional to the intensity of a photobeam.
So, with the formular of intensity loss is

$$I(x) = I_0 \exp(-\frac{x}{X_0})$$

the probability of a single photon travelling uncollided through the crystal is equal to $\exp(-\frac{L}{X_0})$.

$\Rightarrow P(x_{0,BaF_2} = 2{,}03\,cm, L_{BaF_2} = 30\,cm) = 3{,}8 \cdot 10^{-7}$

$P(x_{0,PbWO_4} = 0{,}87\,cm, L_{PbWO_4} = 20\,cm) = 1{,}74 \cdot 10^{-10}$

a+b 3/3

c) As the particle shower is caused by the pair production process where the energy of the poton is used to create electrons and positrons, their mass energy must be at least delivered (1,022 MeV). However this would only lead to not moving particles, which is clearly not the case in a shower.
So, using the energy loss formular $\frac{dE}{dx} = -\frac{E}{X_0}$ for charged particles one could assume the kinetic energy of those particles around 1 MeV which would still lead to a total amount of ca. 250 electrons and 250 positrons at 500 MeV beam energy (or 500 at 1 GeV). In a consecutive shower

however, when the produced charged particles merge again and form a new photon, there is much less energy available. So for every process where there is enough energy left after bremsstrahlung and ionization absorbed energy, there ist most likely only one electron-positron pair produced per photon. 2

(incident?, No!) 0

they do not ⇓ bremsstrahlung,

d) For position resolution it is important that as much of all electromagnetic showers as possible remain within the crystal. Therfore it is needed to know the lateral shower size. The transverse spread of an electromagnetic cascade is characterized by the Molière radius which is given with good approximation by

$$R_M = \frac{21}{E_c(MeV)} X_0$$

On average only 5% of shower energy transpasses the crystal laterally with a crystal-cylinder of radius $2R_M$. X_0 is the radiation length here, which is given for a specific medium (B_aF_2: 2,03 cm, $PbWO_4$: 0,89 cm).
E_c is the critical energy where the rates of energy loss due to bremsstrahlung and ionization are equal and is given by

$$E_c = \frac{550}{Z} MeV$$

for $Z > 12$ by good approximation.
Concluding this I would suggest a cylinder of

$$2R_{M,BaF_2} = 2 \cdot 3{,}12\ cm = 6{,}24\ cm$$

No

$$2R_{M,PbWO_4} = 2 \cdot 1{,}96\ cm = 3{,}92\ cm$$

radius to use to have 95% of all em cascades remaining within the crystal.

c+d Noch

3/5

higher hadron physics   Ho/Ex 3   Julian Bergmann

1, 2

a) $$\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2}\left(\frac{1}{2}\ln\left(\frac{2m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2}\right) - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right)$$

$$T_{max} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2m_e\gamma/M + \left(\frac{m_e}{M}\right)^2} = 2m_e c^2 \beta^2 \gamma^2 \quad \left(\text{for } \frac{2\gamma m_e}{M} \ll 1\right)$$

$$I = (9{,}76\,Z + 58{,}8\,Z^{-0{,}19})\,eV \quad (\text{for } Z \geq 13)$$

$$\frac{\delta}{2} = \ln\left(\frac{\hbar\omega_p}{I}\right) + \ln(\beta\gamma) - \frac{1}{2}$$

$$\hbar\omega_p = \sqrt{\cancel{\#}\rho \cdot (Z/A)} \cdot 28{,}816\,eV = \sqrt{4\pi N_e r_e^3}\,\frac{m_e c^2}{\alpha}$$

$$pV = NRT$$

Approximating the gas as fully consitent of argon (as it's already 90%), we can get the stopping power from fig. 302 on p. 246 in pdg booklet 2012. a little above the curve of iron as they have similar densities.

For $\beta\gamma = 3$ the minimum of $-\frac{dE}{dx}$ against $\beta\gamma$ is reached and $-\frac{dE}{dx} \approx 1{,}5 \frac{MeV}{g \cdot cm^{-2}}$ for all three particles. ✓

With this form of $-\frac{dE}{dx}$, you also have to multiply with argon's density of $1{,}662 \cdot 10^{-3} \frac{g}{cm^3}$, so the formular of total energy loss becomes

$$\Delta E = -\frac{dE}{dx}\,\rho\,\Delta x \qquad 1{,}5\,m\ f \quad -0{,}5$$

with $\Delta x = 4m - 1m = 3m$, as the angle towards the beam is 90°.

Therefor ΔE for $\beta\gamma = 3$ is 375 keV.

For $p = 3\frac{GeV}{c}$, $\frac{dE}{dx}$ has diffrent values depending on the particle:

particle	$-\frac{dE}{dx}$	ΔE
π	$1{,}8 \frac{MeV\,cm^2}{g}$	450 keV
p	$1{,}5 \frac{MeV\,cm^2}{g}$	375 keV
μ	$1{,}9 \frac{MeV\,cm^2}{g}$	480 keV

$1{,}5 \cdot 1{,}662 \cdot 10^{-3} \cdot 300 \approx 750\,keV$

?

correct result with wrong input?

(✓)

2,5/3

b) $\frac{mv^2}{r} \overset{!}{=} qvB \iff r = \frac{p}{qB}$ ✓

$\beta\gamma = \frac{p}{mc} \Rightarrow p = \beta\gamma mc \Rightarrow$ mass dependend r for $\beta\gamma = 3$

particle	m	r
π	$139{,}6 \frac{MeV}{c^2}$	2,79 m ✓
p	$938{,}3 \frac{MeV}{c^2}$	18,78 m ✓
μ	$105{,}7 \frac{MeV}{c^2}$	2,11 m ✓

How did you get the units? ⇒ calculation missing!

$p = 3\frac{GeV}{c} \Rightarrow r = 20{,}01\,m$ ✓ 2/2

beam energy for 500 at 100 ...

c) As shown in a) we can easily differentiate between these particles through energy loss when ~~keeping~~ fixing the same momentum while in b) we can do this with the bending radius in the electromagnetic field by keeping p_T constant. The bending direction in the magnetic field is also different for particle and anti particle. E.g. p is moved into opposite direction as π^- and same as π^+. π^0 isn't bend in the magnetic field at all. The same goes also for $\mu^{\pm}$.

unclear

wrong

μ^0???

So how do you do it?

Exercises 4/ Homework 4

1. For high energy experiments, it is convinient to measure particles momenta in terms of rapidity, defined as $y = \frac{1}{2} \ln \frac{E+p_L}{E-p_L}$. In most cases, it is experimentally less challenging to use the pseudo-rapidity $\eta = -\ln \tan \frac{\vartheta}{2}$.
   (a) Explain why it is "easier" to measure the the pseudo-rapidity compared to the rapidity in an high energy experiment. *(1 points)*
   (b) One feature of rapidities is, that they can be added up (as long as the reference axis is the same). Prove that a particle, which has rapidity a in the CM frame traveling at rapidity b in lab frame, travels with rapidity $c = a + b$ in the lab frame. *(2 points)*
   (c) Compare y and η for protons, (charged) kaons and (charged) pions of $2GeV/c$ momentum for $\eta = 1$, 2 and 5. *(2 points)*
   (d) Prove that $\eta = y$ for large momenta. *(2 points)*

Homework:

1. Under the assumption of no mixing with other pseudoscalar state, for $|\eta>$ and $|\eta'>$ states the following is requiered:

$$X_\eta^2 + Y_\eta^2 = X_{\eta'}^2 + Y_{\eta'}^2 = 1$$

From that, derive these two formulas:

$$X_\eta = Y_{\eta'} = \sqrt{\frac{1}{3}} \cos\theta_p - \sqrt{\frac{2}{3}} \sin\theta_p$$

$$Y_\eta = -X_{\eta'} = -\sqrt{\frac{2}{3}} \cos\theta_p - \sqrt{\frac{1}{3}} \sin\theta_p$$

(3 points)

higher hadron physics Ex/Ho 4 Julius Bergmann

No 1

a) For measurement of rapidity y it is necessary to measure Energy an longitudinal momentum of the particle while in pseudo-rapidity it is only required to get the angle on which the particle was detected regarding a CM-System / reference axis. ✓ 1/1

b)

$$\left(y_{cm} = b = \frac{1}{2}\ln\left(\frac{E+p_L}{E-p_L}\right) \Leftrightarrow e^{2b} = \frac{E+p_L}{E-p_L} \Leftrightarrow e^{2b}(E-p_L) = E+p_L \right.$$

$$\left. \Leftrightarrow \frac{e^{2b}E - E}{-e^{2b}-1} = p_L \quad \Rightarrow \beta = \frac{e^{2b}-1}{-e^{2b}-1} = \frac{p_L}{E} \right)$$

$$q = \begin{array}{l} \\ \text{trans}\{ \\ \\ \text{long.}\{ \end{array} \begin{pmatrix} E \\ p_{t,1} \\ p_{t,2} \\ p_L \end{pmatrix}^T \begin{pmatrix} \gamma & 0 & 0 & +\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ +\beta\gamma & 0 & 0 & \gamma \end{pmatrix} = (E\gamma + p_L\beta\gamma,\ p_{t,1}, p_{t,2},\ E\beta\gamma + p_L\gamma)$$

→ CMS in z-Richtung.

Teilchen in lab: $y = \frac{1}{2}\ln\left(\frac{E\gamma + p_L\beta\gamma + E\beta\gamma + p_L\gamma}{E\gamma + p_L\beta\gamma - E\beta\gamma - p_L\gamma}\right)$ ✓

$$= \frac{1}{2}\ln\left(\frac{E + p_L\beta + E\beta + p_L}{E + p_L\beta - E\beta - p_L}\right) ✓$$

$$= \frac{1}{2}\ln\left(\frac{(1+\beta)(E+p_L)}{(1-\beta)E + (\beta-1)p_L = (1-\beta)(E-p_L)}\right)$$

$$= \frac{1}{2}\ln\left(\frac{(1+\beta)(E+p_L)}{(1-\beta)(E-p_L)}\right) ✓$$

Indizes!!!

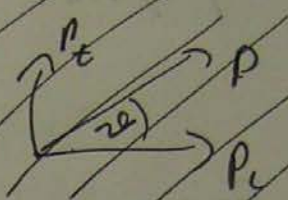
$$= \underbrace{\frac{1}{2}\ln\left(\frac{E+p_L}{E-p_L}\right)}_{\text{Teilchen in CM}} ✓ + \frac{1}{2}\ln\left(\frac{1+\beta}{1-\beta}\right) = \underbrace{\frac{1}{2}\ln\left(\frac{E+p_L}{E-p_L}\right)}_{\text{Teilchen in CM}} + \underbrace{\frac{1}{2}\ln\left(\frac{E+p_L}{E-p_L}\right)}_{\text{CM}} ✓$$

$$= y = c_{\text{Teilchen in Lab}} = a + b$$

what is a, b?

2/2

c) $E^2 = p^2 + m^2 \Rightarrow E = \sqrt{p^2 + m^2}$



$\vartheta = 2\arctan(e^{-\eta})$

$$\tan(\vartheta) = \frac{p_t}{p_l} = \frac{\sqrt{p_l^2 - p^2}}{p_l}$$

$$\Rightarrow p_l = \sqrt{p_l^2 - p^2}\,\tan(2\arctan(e^{-\eta}))$$

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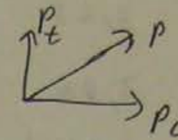
No 1

c) $E^2 = p^2 + m^2 \Rightarrow E = \sqrt{p^2+m^2}$



$\vartheta = 2\arctan(e^{-\eta})$

$\tan(\vartheta) = \frac{p_t}{p_l} = \frac{\sqrt{p^2-p_l^2}}{p_l} \Rightarrow p_l = \tan^{-1}(\vartheta)\sqrt{p^2-p_l^2}$

$\Rightarrow p_l^2 = \left(\tan^{-1}(\vartheta)\sqrt{p^2-p_l^2}\right)^2 = \frac{p^2-p_l^2}{\tan^2(\vartheta)}$

$\Rightarrow p_l^2\left(1+\frac{1}{\tan^2(\vartheta)}\right) = \frac{p^2}{\tan^2(\vartheta)} \Leftrightarrow p_l^2 = p^2\cdot\left(\tan^2(\vartheta)\left(1+\frac{1}{\tan^2(\vartheta)}\right)\right)^{-1}$

$\Rightarrow p_l^2 = p^2\left(\tan^2(\vartheta)+1\right)^{-1} \Rightarrow p_l = \frac{p}{\sqrt{\tan^2(\vartheta)+1}}$

	$E(\frac{GeV}{})$	$p(\frac{GeV}{c})$	y	η	$p_l(\frac{GeV}{c})$
p	2,23	2	0,835	1	1,523
K	2,06	2	1,704	2	1,928
π	2,0048	2	3,340	5	1,9998

see the rest of the table on the back ⇒

d) $\tan\left(\frac{x}{2}\right) = \frac{\tan(x)}{1+\sqrt{1+\tan^2(x)}}$

$\eta = -\ln\left(\tan\left(\frac{\vartheta}{2}\right)\right) = -\ln\left(\frac{\frac{\sqrt{p^2-p_l^2}}{p_l}}{1+\sqrt{1+\left(\frac{\sqrt{p^2-p_l^2}}{p_l}\right)^2}}\right)$

$= -\ln\left(\frac{\sqrt{p^2-p_l^2}}{p_l}\cdot\left(1+\frac{1}{p_l}\sqrt{p_l^2+p^2-p_l^2}\right)^{-1}\right) = -\ln\left(\frac{\sqrt{p^2-p_l^2}}{p_l}\left(\frac{p_l+p}{p_l}\right)^{-1}\right)$

$= -\ln\left(\frac{\sqrt{p^2-p_l^2}}{p_l+p}\right) = -\ln\left(\frac{\sqrt{p-p_l}\sqrt{p+p_l}}{\sqrt{p+p_l}\sqrt{p+p_l}}\right) = -\ln\left(\frac{\sqrt{p-p_l}}{\sqrt{p+p_l}}\right)$

$= \ln\left(\frac{\sqrt{p+p_l}}{\sqrt{p-p_l}}\right) = \frac{1}{2}\ln\left(\frac{p+p_l}{p-p_l}\right)$

$E \approx |p| \Rightarrow y = \frac{1}{2}\ln\left(\frac{E+p_l}{E-p_l}\right) \approx \frac{1}{2}\ln\left(\frac{p+p_l}{p-p_l}\right) = \eta$

2/2

	E (GeV)	$p\left(\frac{GeV}{c}\right)$	y	η	$p_\perp\left(\frac{GeV}{c}\right)$
p	2,23	2	1,311	2	1,928
K	2,06	2	2,105	5	1,9998
π	2,0043	2	0,995	1	1,523
p	2,23	2	1,455	5	1,9998
K	2,06	2	0,949	1	1,523
π	2,0043	2	1,968	2	1,928

2/2

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Homework 1

$$|\eta\rangle = X_\eta \frac{1}{\sqrt{2}}|u\bar{u}+d\bar{d}\rangle + Y_\eta |s\bar{s}\rangle$$

$$|\eta'\rangle = X_{\eta'} \frac{1}{\sqrt{2}}|u\bar{u}+d\bar{d}\rangle + Y_{\eta'} |s\bar{s}\rangle$$

$$|\eta_8\rangle = \frac{1}{\sqrt{6}}|u\bar{u}+d\bar{d}-2s\bar{s}\rangle$$

$$|\eta_0\rangle = \frac{1}{\sqrt{3}}|u\bar{u}+d\bar{d}+s\bar{s}\rangle$$

$$|\eta\rangle = \cos(\theta_P)|\eta_8\rangle - \sin(\theta_P)|\eta_0\rangle$$

$$|\eta'\rangle = \sin(\theta_P)|\eta_8\rangle + \cos(\theta_P)|\eta_0\rangle$$

$$|\eta\rangle = X_\eta \frac{1}{\sqrt{2}}|u\bar{u}+d\bar{d}\rangle + Y_\eta|s\bar{s}\rangle = \cos(\theta_P)|\eta_8\rangle - \sin(\theta_P)|\eta_0\rangle$$

$$= \cos(\theta_P)\cdot\frac{1}{\sqrt{6}}|u\bar{u}+d\bar{d}-2s\bar{s}\rangle - \sin(\theta_P)\cdot\frac{1}{\sqrt{3}}|u\bar{u}+d\bar{d}+s\bar{s}\rangle$$

$$= \underbrace{\left(\cos(\theta_P)\cdot\frac{1}{\sqrt{6}} - \sin(\theta_P)\cdot\frac{1}{\sqrt{3}}\right)}_{X_{\eta'}\frac{1}{\sqrt{2}}}|u\bar{u}+d\bar{d}\rangle + \underbrace{\left(\frac{-2}{\sqrt{6}}\cos(\theta_P) - \frac{1}{\sqrt{3}}\sin(\theta_P)\right)}_{Y_\eta}|s\bar{s}\rangle$$

$$\Rightarrow Y_\eta = -\sqrt{\frac{2}{3}}\cos(\theta_P) - \frac{1}{\sqrt{3}}\sin(\theta_P)$$

$$X_\eta = \sqrt{\frac{1}{3}}\cos(\theta_P) - \sqrt{\frac{2}{3}}\sin(\theta_P)$$

$$|\eta'\rangle = X_{\eta'}\cdot\frac{1}{\sqrt{2}}|u\bar{u}+d\bar{d}\rangle + Y_{\eta'}|s\bar{s}\rangle = \sin(\theta_P)|\eta_8\rangle + \cos(\theta_P)|\eta_0\rangle$$

$$= \sin(\theta_P)\cdot\frac{1}{\sqrt{6}}|u\bar{u}+d\bar{d}-2s\bar{s}\rangle + \cos(\theta_P)\cdot\frac{1}{\sqrt{3}}|u\bar{u}+d\bar{d}+s\bar{s}\rangle$$

$$= \underbrace{\left(\sin(\theta_P)\cdot\frac{1}{\sqrt{6}} + \cos(\theta_P)\frac{1}{\sqrt{3}}\right)}_{X_{\eta'}\frac{1}{\sqrt{2}}}|u\bar{u}+d\bar{d}\rangle + \underbrace{\left(\frac{-2}{\sqrt{6}}\sin(\theta_P) + \frac{1}{\sqrt{3}}\cos(\theta_P)\right)}_{Y_{\eta'}}|s\bar{s}\rangle$$

$$\Rightarrow X_{\eta'} = \sqrt{\frac{2}{3}}\cos(\theta_P) + \sqrt{\frac{1}{3}}\sin(\theta_P) = -Y_\eta$$

$$Y_{\eta'} = -\sqrt{\frac{2}{3}}\sin(\theta_P) + \frac{1}{\sqrt{3}}\cos(\theta_P) = X_\eta$$

3/3

Homework 5

1. Exotic quantum numbers: Prove that the quantum numbers 0^{+-}, 1^{-+} and 2^{+-} are not allowed for mesons (quark-antiquark). Hint: Start with the allowed cases for the spin and then, for a fixed spin, check the cases for J. *(5 points)*

2. Which of the following decays meson are allowed in strong interaction? Check the known rules (parity, isospin, ...) and state which are violated in the decay. *(5 points)*

   (a) $\rho \to \pi^+\pi^-$ and $\omega \to \pi^+\pi^-$

   (b) $\rho \to \pi^0\pi^0$ and $\omega \to \pi^0\pi^0$

   (c) $\rho \to \eta\pi^0$ and $\omega \to \eta\pi^0$

   (d) $\rho^+ \to \eta\pi^+$

   (e) $J/\psi \to \pi^0\pi^0$ and $J/\psi \to \pi^+\pi^-$

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No 1

0^{+-} : spin 0 or 1.

Spin 0: $J=0,\ S=0,\ J=L\oplus S \Rightarrow L=0,\ P=(-1)^{L+1}=-1$ ↯

Spin 1: $J=0,\ S=1,\ J=L\oplus S \Rightarrow L=0$ or 1

$L=0: P=(-1)^{L+1}=-1$ ↯

$L=1: P=(-1)^{L+1}=1,\ C=(-1)^{L+S}=1$ ↯

1^{-+} : Spin 0: $J=1,\ S=0,\ J=L\oplus S \Rightarrow L=1$

$L=1: P=(-1)^{L+1}=+1,\ C=(-1)^{L+S}=-1$ ↯

Spin 1: $J=1,\ S=1,\ J=L\oplus S \Rightarrow L=0$ or 1 or 2

$L=0: P=(-1)^{L+1}=-1,\ C=(-1)^{L+S}=-1$ ↯

$L=1: P=(-1)^{L+1}=1$ ↯

$L=2: P=(-1)^{L+1}=-1,\ C=(-1)^{L+S}=-1$ ↯

2^{+-} : Spin 0: $J=2,\ S=0,\ J=L\oplus S \Rightarrow L=2$

$L=0: P=(-1)^{L+1}=-1$ ↯

Spin 1: $J=2,\ S=1,\ J=L\oplus S \Rightarrow L=1$ or 2 or 3

$L=1: P=(-1)^{L+1}=1,\ C=(-1)^{L+S}=1$ ↯

$L=2: P=(-1)^{L+1}=-1$ ↯

$L=3: P=(-1)^{L+1}=1,\ C=(-1)^{L+S}=1$ ↯

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No 2

a) $\rho \to \pi^+\pi^-$: charge: $0 = 1 - 1$

parity: $-1 = (-1)\cdot 1 \cdot 1 \cdot (-1)^l$

ang. mom.: $\underbrace{1}_{J_\rho} \oplus \underbrace{1}_{s_\rho} = \underbrace{0}_{J_{\pi^+}} + \underbrace{0}_{J_{\pi^-}} + \underbrace{0}_{l}$ } $l = 1$ f

Isospin: $1 = 1 \oplus 1$ ✓

I_z : $0 = 1 - 1$ ⇒ permitted

G-Parity: $1 = (-1)(-1)$ ✓

$\omega \to \pi^+\pi^-$: Isospin: $1 = 1 \oplus 1$ ⇒ not allowed

G-Parity: $-1 \neq (-1)(-1)$

(rest similar to ρ)

b) $\rho \to \pi^0\pi^0$: charge: $0 = 0 + 0$

parity: $-1 = (-1)(-1)(-1)^1$ ✓

ang. mom.: $1 \oplus 1 = 0 + 0 + 1$ ⇒ not allowed

C-Parity: $-1 \neq 1 \cdot 1$ ✓

CP-viol.: $1 = (-1)(-1)$

Isospin: $1 = 1 \oplus 1$

I_z : $0 = 0 + 0$

G-Parity: $1 = (-1)(-1)$

$\omega \to \pi^0\pi^0$: Isospin: $0 = 1 \oplus 1$ ⇒ not allowed

G-Parity: $-1 \neq (-1)(-1)$ ✓

C-Parity: $-1 \neq 1 \cdot 1$ ✓

c) $\rho \to \eta\pi^0$: charge: $0 = 0 + 0$

parity: $-1 = 1 \cdot (-1) \cdot 1$

ang. mom.: $1 \oplus 1 = 0 + 0 + 0$ } ff ⇒ not allowed

C-Parity: $-1 \neq 1 \cdot 1$ ✓

CP-viol.: $1 \neq 1 \cdot (-1)$

Isospin: $1 = 1 \oplus 0$

I_z : $0 = 0 + 0$

G-Parity: $1 \neq (-1) \cdot 1$ ✓

$\omega \to \eta\pi^0$: Isospin: $0 \neq 1 \oplus 0$ ✓ ⇒ not allowed

G-Parity: $(-1) = (-1) \cdot 1$

C-Parity: $-1 \neq 1 \cdot 1$ ✓

CP-viol.: $1 \neq 1 \cdot (-1)$

d) $\rho^+ \to \eta\pi^+$: charge: $1 = 0 + 1$ ✓

parity: $-1 = (-1)(-1)(-1)^1$ $-0,5$

ang. mom.: $1 \oplus 1 = 0 + 0 + 1$ ⇒ not allowed

Isospin: $1 \neq 0 \oplus 1$ f

I_z : $1 = 0 + 1$

G-parity: $1 \neq 1 \cdot (-1)$ ✓

c) $J/\psi \to \pi^0\pi^0$:

charge : $0 = 0 + 0$

parity : $-1 = 1 \cdot 1 \cdot (-1)^1$ ff

ang. mom. : $1 \oplus 1 = 0 + 0 + 1$

C-Parity : $-1 \neq (-1)(-1)$ ✓ ⇒ not allowed ✓

CP-viol. : $1 = (-1)(-1)$

Isospin : $0 = 1 \oplus 1$

I_z : $0 = 0 + 0$

G-Parity : $-1 \neq (-1)(-1)$ ✓

$J/\psi \to \pi^+\pi^-$:

charge : $0 = 1 - 1$

parity : $-1 = (-1) \cdot 1 \cdot 1$ ff

ang. mom. : $1 \oplus 1 = 0 + 0 + 0$

G-Parity : $-1 \neq (-1)(-1)$ ✓ ⇒ not allowed

Isospin : $0 = 1 \oplus 1$

I_z : $0 = 1 - 1$

~~3~~ 5/5

−1 for P and J being wrong several times