Hadron and Nuclear Physics Julian Bergmann

Problem 1.1

$m_{\Delta^{++}} = 1232 \frac{MeV}{c^2}$, $m_\pi = 139{,}570 \frac{MeV}{c^2}$, $m_p = 938{,}272 \frac{MeV}{c^2}$

particle at rest: $E_G = m_{\Delta^{++}} c^2$, $E_\pi + E_p = E_G$, $|\vec{p}_\pi| = |\vec{p}_p|$

$$\Rightarrow \quad 0 = |\vec{p}_\pi| - |\vec{p}_p| = \frac{1}{c}\left(\sqrt{E_\pi^2 - m_\pi^2 c^4} - \sqrt{E_p^2 - m_p^2 c^4}\right)$$

$$= \frac{1}{c}\left(\sqrt{E_\pi^2 - m_\pi^2 c^4} - \sqrt{(E_G - E_\pi)^2 - m_p^2 c^4}\right)$$

$$\Leftrightarrow \quad E_\pi^2 - m_\pi^2 c^4 = E_G^2 - 2E_G E_\pi + E_\pi^2 - m_p^2 c^4$$

$$\Leftrightarrow \quad 2E_G E_\pi = E_G^2 + m_\pi^2 c^4 - m_p^2 c^4$$

$$\Leftrightarrow \quad E_\pi = \frac{1}{2}\left(E_G + \frac{m_\pi^2 c^2}{m_{\Delta^{++}}} - \frac{m_p^2 c^2}{m_{\Delta^{++}}}\right)$$

$$= \frac{1}{2}\left(1232\, MeV + \frac{(139{,}57)^2}{1232} MeV - \frac{(938{,}272)^2}{1232} MeV\right)$$

$$= 266{,}619\ MeV$$

$$E_p = \frac{1}{2}\left(E_G + \frac{m_p^2 c^2}{m_{\Delta^{++}}} - \frac{m_\pi^2 c^2}{m_{\Delta^{++}}}\right)$$

$$= 965{,}381\ MeV$$

Relativist.!

$$|\vec{p}_\pi| = \frac{1}{c}\sqrt{E_\pi^2 - m_\pi^2 c^4} = 227{,}169 \frac{MeV}{c} = |\vec{p}_p|$$

$$E_{kin,\pi} = \frac{|\vec{p}_\pi|^2}{2 m_\pi} = 184{,}875\ MeV$$ f

$$E_{kin,p} = \frac{|\vec{p}_p|^2}{2 m_p} = 27{,}5005\ MeV$$ (r)

$$\beta_\pi = \frac{|\vec{p}_\pi| c}{E_\pi} = \frac{227{,}169\ MeV}{266{,}619\ MeV} = 0{,}852037$$

$$\beta_p = \frac{|\vec{p}_p| c}{E_p} = \frac{227{,}169\ MeV}{965{,}381\ MeV} = 0{,}235316$$

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Problem 1.2

a) $p + p \to \pi^+ + \pi^0$ ✗ electrical charge violated $(1+1 = 1+0)$

b) $\eta \to \gamma + \gamma$ ✓ EM-interaction. (Parity $(-1) \to (-1)\cdot(-1)\cdot(-1)^{L=1}$)

Schw. f c) $\Sigma^0 \to \Lambda + \pi^0$ ✗ Parity violated $((+1) \to (+1)\cdot(-1))$

d) $\Sigma^- \to n + \pi^-$ ✓ weak interaction: $(dds \to udd + \bar{u}d)$

e) $e^+ + e^- \to \mu^+ + \mu^-$ ✓ EM-interaction, but much Energy needed!

f) $\mu^- \to e^- + \bar{\nu}_e$ ✗ Lepton number violated. $(L: 1 \to 1 - 1)$

g) $\Delta^+ \to p + \pi^0$ ✓ strong interaction $(J: \frac{3}{2} \to \frac{1}{2} + 0 + 1\ (\in L))$

h) $\bar{\nu}_e + p \to n + e^+$ ✓ weak interaction $(uud \to udd)$

i) $e^- + p \to \nu_e + \pi^0$ ✗ Baryon number violated $(B: 0+1 \to 0+0)$

f j) $p+p \to \Sigma^+ + n + K^0 + \pi^+ + \pi^0$ ✗ Parity violated (Parity: $(+1)^2 \to (+1)^2(-1)^3$)

k) $p \to e^+ + \gamma$ ✗ Baryon number violated $(B: 1 \to 0+0)$

l) $p+p \to p+p+p+\bar{p}$ ✓ strong interaction (+Energy)

m) $n + \bar{n} \to \pi^+ + \pi^- + \pi^0$ ✓ strong interaction

n) $\pi^+ + n \to \pi^- + p$ ✗ electrical charge violated $(+1+0 \to -1+1)$

o) $K^- \to \pi^- + \pi^0$ ✓ weak interaction $(s\bar{u} \to d\bar{u} + \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}))$

p) $\Sigma^+ + n \to \Sigma^- + p$ ✗ electrical charge violated $(+1+0 \to -1+1)$

q) $\Sigma^0 \to \Lambda + \gamma$ ✓ EM-interaction (Parity: $(+1) \to (+1)(-1)(-1)^{L=1}$)

r) $\Xi^- \to \Lambda + \pi^-$ ✓ weak interaction $(dss \to uds + d\bar{u})$

s) $\Xi^0 \to p + \pi^-$ ✗ $\Delta S = 2 \Rightarrow$ highly suppressed (10^{-8}) weak I.

t) $\pi^- + p \to \Lambda + K^0$ ✓ strong interaction $(d\bar{u} + uud \to uds + d\bar{s})$

u) $\pi^0 \to \gamma + \gamma$ ✓ EM-interaction (Parity: $(-1) \to (-1)(-1)(-1)^{L=1}$)

v) $\Sigma^- \to n + e^- + \bar{\nu}_e$ ✓ weak interaction $(dds \to udd)$

f w) $n \to p + \mu^- + \bar{\nu}_\mu$ ✓ weak interaction (high Energy needed)

x) $\Delta^+ \to p$ ✗ angular momentum violated $(J: \frac{3}{2} \to \frac{1}{2})$

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Problem 1.3

a) $\pi^-: |1,-1\rangle$, $p: |\frac{1}{2},\frac{1}{2}\rangle$

$|\pi^- p\rangle \hat{=} |1,-1\rangle|\frac{1}{2},\frac{1}{2}\rangle \hat{=} \sqrt{\frac{1}{3}}|\frac{3}{2},-\frac{1}{2}\rangle - \sqrt{\frac{2}{3}}|\frac{1}{2},-\frac{1}{2}\rangle$

$K^0: \langle\frac{1}{2},-\frac{1}{2}|$, $\Sigma^0: \langle 1,0|$

$\langle\Sigma^0 K^0| \hat{=} \langle 1,0|\langle\frac{1}{2},-\frac{1}{2}| \hat{=} \sqrt{\frac{2}{3}}\langle\frac{3}{2},-\frac{1}{2}| + \sqrt{\frac{1}{3}}\langle\frac{1}{2},-\frac{1}{2}|$

$\Rightarrow \langle\Sigma^0 K^0|T|\pi^- p\rangle = \frac{\sqrt{2}}{3}\langle\frac{3}{2},-\frac{1}{2}|T|\frac{3}{2},-\frac{1}{2}\rangle - \frac{\sqrt{2}}{3}\langle\frac{1}{2},-\frac{1}{2}|T|\frac{1}{2},-\frac{1}{2}\rangle$

$= \frac{\sqrt{2}}{3}T_{3/2} - \frac{\sqrt{2}}{3}T_{1/2} = M_a$

b) $|\pi^- p\rangle \hat{=} \sqrt{\frac{1}{3}}|\frac{3}{2},-\frac{1}{2}\rangle - \sqrt{\frac{2}{3}}|\frac{1}{2},-\frac{1}{2}\rangle$

$K^+: \langle\frac{1}{2},\frac{1}{2}|$, $\Sigma^-: \langle 1,-1|$

$\langle\Sigma^- K^+| \hat{=} \langle 1,-1|\langle\frac{1}{2},\frac{1}{2}| \hat{=} \sqrt{\frac{1}{3}}\langle\frac{3}{2},-\frac{1}{2}| - \sqrt{\frac{2}{3}}\langle\frac{1}{2},-\frac{1}{2}|$

$\Rightarrow \langle\Sigma^- K^+|T|\pi^- p\rangle = \frac{1}{3}\langle\frac{3}{2},-\frac{1}{2}|T|\frac{3}{2},-\frac{1}{2}\rangle + \frac{2}{3}\langle\frac{1}{2},-\frac{1}{2}|T|\frac{1}{2},-\frac{1}{2}\rangle$

$= \frac{1}{3}T_{3/2} + \frac{2}{3}T_{1/2} = M_b$

c) $\pi^+: |1,1\rangle$, $p: |\frac{1}{2},\frac{1}{2}\rangle$, $K^+: \langle\frac{1}{2},\frac{1}{2}|$, $\Sigma^+: \langle 1,1|$

$|\pi^+ p\rangle \hat{=} |1,1\rangle|\frac{1}{2},\frac{1}{2}\rangle \hat{=} |\frac{3}{2},\frac{3}{2}\rangle$

$\langle\Sigma^+ K^+| \hat{=} \langle 1,1|\langle\frac{1}{2},\frac{1}{2}| \hat{=} \langle\frac{3}{2},\frac{3}{2}|$

$\Rightarrow \langle\Sigma^+ K^+|T|\pi^+ p\rangle = \langle\frac{3}{2},\frac{3}{2}|T|\frac{3}{2},\frac{3}{2}\rangle$

$= T_{3/2} = M_c$

$\Rightarrow$ branching ratios: $\sigma_i \sim M_i^2$

	a)	b)	c)
$I=\frac{3}{2}$ dominant $\Rightarrow T_{1/2}=0$:	$\frac{2}{9}$:	$\frac{1}{9}$:	1
$I=\frac{1}{2}$ dominant $\Rightarrow T_{3/2}=0$:	$\frac{2}{9}$:	$\frac{4}{9}$:	0

$\hat{=}$

	a) b) c)
$I=3/2$:	2 : 1 : 9
$I=1/2$:	1 : 2 : 0

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