Höhere Hadronenphysik

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Problem 5.1

momentum conservation:

$$K^- + p \rightarrow \pi^0 + \Lambda \qquad \Rightarrow \quad p_{K^-} = p_{\pi^0} =: p$$

(p: rest, Λ: rest)

energy conservation:

$$E_{K^-} + E_p = E_{\pi^0} + E_\Lambda$$

$$\Leftrightarrow \sqrt{p^2 + m_{K^-}^2} + m_p \overset{!}{=} \sqrt{p^2 + m_{\pi^0}^2} + m_\Lambda$$

solved with mathematica:

$$p = \sqrt{\frac{(m_{K^-} - m_p - m_{\pi^0} - m_\Lambda)(m_{K^-} + m_p + m_{\pi^0} + m_\Lambda)(m_{K^-} - m_p - m_{\pi^0} + m_\Lambda)(m_{K^-} - m_p + m_{\pi^0} + m_\Lambda)}{4(\;_p - \;_\Lambda)^2}}$$

entering following values

$m_K = 493{,}7$ MeV , $m_p = 938{,}3$ MeV , $m_{\pi^0} = 135{,}0$ MeV

$m_\Lambda = 1115{,}7$ MeV

causes p to have the value of

$$p = 529{,}99 \text{ MeV}$$

which leads to an incidental total Energy of

$$E_{K^-} = \sqrt{p^2 + m_{K^-}^2} = 724{,}31 \text{ MeV} .$$

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Problem 5.2

a) The P state would mean that the deuteron's Parity, which can be calculated using $(-1)^L = (-1)^1 = -1$, would be negative.
As the Deuteron has purely positive Parity, that is the reason why the P state isn't allowed.

b) Unlike the P state, the G state ($L=4$) would lead to a positive Parity and isn't forbidden for that reason. However for quantum numbers it is necessary to fullfill also $|L-S| \le J \le |L+S|$.
With $L=4$ and $J=1$ this equotation would be $|4-S| \le 1 \le |4+S|$, which is only fulfillable with $S \ge 3$. But as the deuteron is a bound state of a proton and a neutron which both have Spin $\frac{1}{2}$, such a high spin cannot be reached by the deuteron and therefor the G state cannot contribute.

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Problem 5.3

a) $J(\pi) = 0$

$J(d) = 1$

$\Rightarrow J(n) = 1$

b) $S(\pi) = 0$

$S(d) = 1$

$S(n) = 1/2$

$\Rightarrow$ total $S = 1$

c) As the pion doesn't have spin, having the deuteron a spin opposite to the spin of both neutrons would mean to totally flip the complete spin during the reaction, which is highly unlikely.

Using quantum mechanics we can use Clebsch Gordan to observe the spin-states during the reaction:

$$\langle S_{nn}, S_{nn,z} | S_d, S_{d_z} \rangle \Leftrightarrow (\langle S_{n_1}, S_{n_{1,z}} | \langle S_{n_2}, S_{n_{2,z}} |) | 1, 1 \rangle$$

$$\Leftrightarrow (\langle \tfrac{1}{2}, -\tfrac{1}{2} | \langle \tfrac{1}{2}, -\tfrac{1}{2} |) | 1, 1 \rangle$$

we set the deuteron's spin direction to +1, so for the neutron's we need negative spin-direction in z-component.

$$\langle \tfrac{1}{2}, -\tfrac{1}{2} | \langle \tfrac{1}{2}, -\tfrac{1}{2} | \underset{C.G.}{\Leftrightarrow} \langle 1, -1 |$$

$$\Rightarrow \langle 1, -1 | 1, 1 \rangle = 0$$

So it is not possible to have the neutron have a spin opposite to the total spin before the reaction.