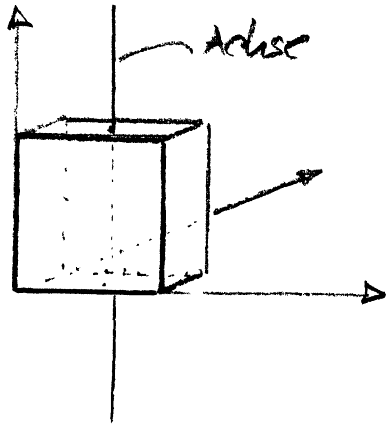
H 9

$$I = \rho \int d^2 \, dV$$

a)



$$I = \rho \int_0^a dx \int_0^a dy \int_0^a dz \; (x - a/2)^2 + (y - a/2)^2$$

$$= \rho \int dV \, (x^2 + y^2) + 2\rho \int dV \, \frac{a^2}{4} - 2\rho \int dV \, 2 \frac{a}{2} x$$

$$= \frac{2}{3} M a^2 + \frac{a^2}{2} M - a^2 M$$

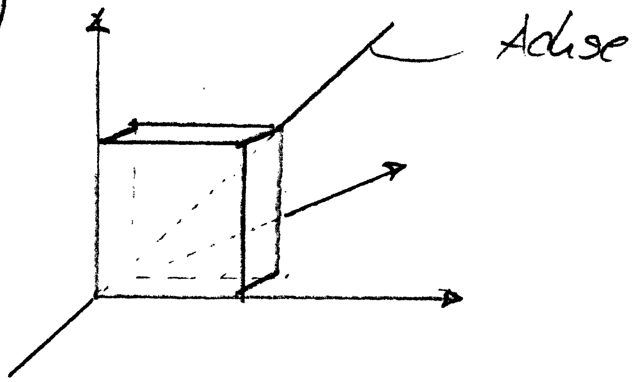
$$= \frac{2}{3} M a^2 - \frac{a^2}{2} M$$

$$= R^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2$$

$$= \frac{1}{6} M a^2$$

H9)

5)



Abstand zur Achse $\quad d^2 = (x-c)^2 + (y-c)^2 + (z-c)^2$

Minimal für $\quad c = \frac{1}{3}(x+y+z)$

$$I = \rho \int_0^a dx \int_0^a dy \int_0^a dz \; (x-c)^2 + (y-c)^2 + (z-c)^2$$

$$= 3\rho \int dx \int dy \int dz \; \left(x - \frac{1}{3}(x+y+z)\right)^2$$

$$= 3\rho \int dx \int dy \int dz \; \left(\frac{2}{3}x - \frac{1}{3}y - \frac{1}{3}z\right)^2$$

$$= 3\rho \int dy \int dz \; \frac{4a^3}{27} - 2\frac{a^2(y+z)}{9} + \frac{a}{9}(y+z)^2$$

$$= \frac{4a^5}{9}\rho - \frac{2}{3}\rho a^2 \int dy \int dz \,(y+z) + \frac{1}{3}a\rho \int dy \int dz \,(y+z)^2$$

$$= \frac{4a^5\rho}{9} - \frac{2}{3}\rho a^2 \cdot a^3 + \frac{1}{3}a\rho \cdot \frac{7a^4}{6}$$

$$= \frac{1}{6}a^5\rho$$

$$= \frac{1}{6}Ma^2$$

Der Würfel besitzt eine Kugel als Trägheitsellipsoid. Also ist das Trägheitsmoment für alle Achsen durch den Schwerpunkt gleich.

H10|

Trägheitsmoment der Masse: I_M

$$L_z = I_M \omega_0$$

Satz von Steiner

$$I(t) = I_M + M r^2(t) \quad , \; r(t) = v \cdot t$$

$$= I_M + M v^2 t^2$$

$$\rightarrow L_z = I_M \omega_0 = I(t)\, \omega(t)$$

$$\rightarrow \omega(t) = \omega_0 \frac{I_M}{I_M + M v^2 t^2}$$