$L = (q_1, q_2, \dot{q}_1, \dot{q}_2, t)$ $\qquad$ $\frac{d}{dt}\delta q = \delta \dot{q}$

$\boxed{P15}$ $\delta q_i(t_1) = \delta q_i(t_2) = 0$

$\delta S = S[q+\delta q] - S[q]$

Hamilton'sches Prinzip: $\delta S = \delta \int_{t_1}^{t_2} L\, dt = 0$

$$\Rightarrow \delta S = \int_{t_1}^{t_2} \left(L(q_1+\delta q_1, q_2+\delta q_2, \dot{q}_1+\delta\dot{q}_1, \dot{q}_2+\delta\dot{q}_2, t) - L(q_1, q_2, \dot{q}_1, \dot{q}_2, t)\right) dt = 0$$

$$= \int_{t_1}^{t_2} \left(L(q_1, q_2, \dot{q}_1, \dot{q}_2, t) + \frac{\partial L}{\partial q_1}\delta q_1 + \frac{\partial L}{\partial q_2}\delta q_2 + \frac{\partial L}{\partial \dot{q}_1}\delta \dot{q}_1 + \frac{\partial L}{\partial \dot{q}_2}\delta \dot{q}_2 - L(q_1, q_2, \dot{q}_1, \dot{q}_2, t)\right) dt = 0$$

$$= \int_{t_1}^{t_2} \left(\frac{\partial L}{\partial q_1}\delta q_1 + \frac{\partial L}{\partial q_2}\delta q_2 + \frac{\partial L}{\partial \dot{q}_1}\delta \dot{q}_1 + \frac{\partial L}{\partial \dot{q}_2}\delta \dot{q}_2 \right) dt = 0$$

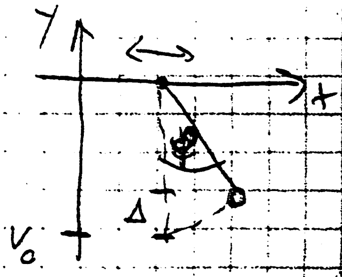
part. Integration: $\int_{t_1}^{t_2} \frac{\partial L}{\partial \dot{q}}\delta\dot{q}\, dt = \int_{t_1}^{t_2} \frac{\partial L}{\partial \dot{q}} \frac{d}{dt}\delta q = -\int_{t_1}^{t_2} \frac{d}{dt}\frac{\partial L}{\partial \dot{q}}\delta q + \underbrace{\frac{\partial L}{\partial \dot{q}}\delta q\Big|_{t_1}^{t_2}}_{=0}$

$$= \int_{t_1}^{t_2} \left(\frac{\partial L}{\partial q_1}\delta q_1 + \frac{\partial L}{\partial q_2}\delta q_2 - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_1}\delta q_1 - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_2}\delta q_2 \right) dt + \underbrace{\frac{\partial L}{\partial \dot{q}_1}\delta q_1\Big|_{t_1}^{t_2} + \frac{\partial L}{\partial \dot{q}_2}\delta q_2\Big|_{t_1}^{t_2}}_{=0} = 0$$

$$= \int_{t_1}^{t_2} \left(\left(\frac{\partial L}{\partial q_1} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_1}\right)\delta q_1 + \left(\frac{\partial L}{\partial q_2} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_2}\right)\delta q_2 \right) dt = 0$$

Aufgabe P16

Aufhängepunkt: $x_0(t) = A \cdot \sin \Omega t$



Masse: $x(t) = x_0(t) + l \cdot \sin\varphi$

$y(t) = l \cdot \cos\varphi$

$\dot{x}(t) = A\Omega\cos\Omega t + l\dot{\varphi}\cos\varphi \qquad \dot{y}(t) = -l\dot{\varphi}\sin\varphi$

$$T = \frac{m}{2}v^2 = \frac{m}{2}(\dot{x}^2+\dot{y}^2) = \frac{m}{2}\Big(A^2\Omega^2\cos^2\Omega t + \underline{l^2\dot{\varphi}^2\cos^2\varphi} + \cancel{2A\Omega} +$$
$$+ 2Al\Omega\dot{\varphi}\cos\Omega t\cos\varphi + \underline{l^2\dot{\varphi}^2\sin^2\varphi}\Big) =$$
$$= \frac{m}{2}\Big(A^2\Omega^2\cos^2\Omega t + l^2\dot{\varphi}^2 + 2Al\Omega\cos\Omega t\,\dot{\varphi}\cos\varphi\Big)$$

$V = V_0 + mg\Delta \qquad \Delta = l - y(t) = l - l\cos\varphi = l(1-\cos\varphi)\ ;\ V_0 = -mgl$

$V = -mgl + mgl(1-\cos\varphi) = -mgl\cos\varphi$

$$L = T - V = \frac{m}{2}\Big(A^2\Omega^2\cos^2\Omega t + l^2\dot{\varphi}^2 + 2Al\Omega\cos\Omega t\,\dot{\varphi}\cos\varphi + 2gl\cos\varphi\Big)$$

$$\frac{\partial L}{\partial\varphi} = m\big(\underline{-Al\Omega\cos\Omega t\,\dot{\varphi}\sin\varphi} - gl\sin\varphi\big)$$

$$\frac{\partial L}{\partial\dot{\varphi}} = ml^2\dot{\varphi} + mAl\Omega\cos\Omega t\cos\varphi$$

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\varphi}} = ml^2\ddot{\varphi} - mAl\Omega^2\sin\Omega t\cos\varphi - \underline{mAl\Omega\cos\Omega t\sin\varphi\cdot\dot{\varphi}}$$

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\varphi}} - \frac{\partial L}{\partial\varphi} = 0 \iff l^2\ddot{\varphi} - Al\Omega^2\sin\Omega t\cos\varphi + gl\sin\varphi = 0$$

$$\ddot{\varphi} + \frac{g}{l}\sin\varphi - \frac{A\Omega^2}{l}\sin\Omega t\cos\varphi = 0$$

Für kleine Auslenkungen gilt: $\sin\varphi \approx \varphi$, $\cos\varphi \approx 1$ $\Rightarrow \ddot{\varphi} + \frac{g}{l}\varphi = \frac{A\Omega^2}{l}\sin\Omega t$

Bekannte (inhomogene) Dgl. für erzwungene Schwingungen

Allg. Lsg. d. hom. Dgl: $\ddot{\varphi} + \frac{g}{l}\varphi = 0$ ist: $\varphi(t) = \varphi_0 \cdot \sin(\omega_0 t + \delta)$ mit $\omega_0 = \sqrt{g/l}$

Spezielle Lsg. d. inhom. Dgl: Versuch: $\varphi = C\cdot\sin\Omega t \qquad \ddot{\varphi} = -C\Omega^2\sin\Omega t$

einsetzen: $-C\Omega^2\cancel{\sin}\,\Omega t + \frac{g}{l}C\cancel{\sin}\,\Omega t = \frac{A\Omega^2}{l}\cancel{\sin}\,\Omega t\ ;\ C\left(\frac{g}{l} - \Omega^2\right) = \frac{A\Omega^2}{l}$

$C = \frac{A}{l}\frac{\Omega^2}{g/l - \Omega^2} = \frac{A}{l}\frac{\Omega^2}{\omega_0^2 - \Omega^2}$; Anfangsbed.: $x(0) = 0 \iff \delta = 0$

$\dot{x}(0) = 0 \iff \varphi_0 = -\frac{A}{l}\frac{\Omega}{\omega_0}\frac{\Omega^2}{\omega_0^2 - \Omega^2}$

$$\varphi(t) = \varphi_0\sin(\omega_0 t + \delta) + \frac{A}{l}\frac{\Omega^2}{\omega_0^2 - \Omega^2}\sin\Omega t$$

$$\varphi(t) = \frac{A}{l}\frac{\Omega^2}{\omega_0^2 - \Omega^2}\left(\sin\Omega t - \frac{\Omega}{\omega_0}\sin\omega_0 t\right)$$