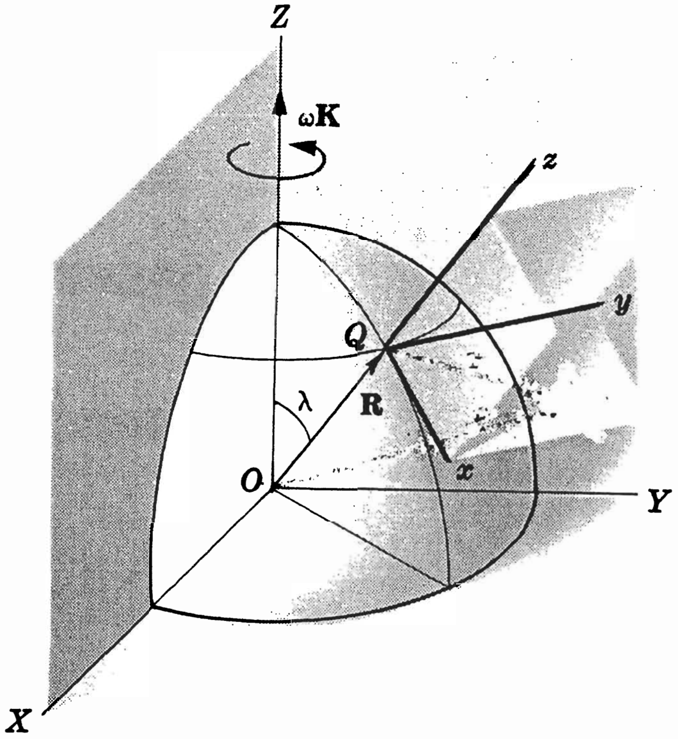
P17)



Im bewegten System: $\vec{\omega} = \omega \vec{e}_Z = -\omega \sin\lambda \, \vec{e}_x{}' + \omega \cos\lambda \, \vec{e}_z{}'$

$$= -\omega \cos\theta \, \vec{e}_x{}' + \omega \sin\theta \, \vec{e}_z{}'$$

$$\left[\begin{array}{l} \sin(90° - \theta) = \cos(-\theta) = \cos\theta \\ \cos(90° - \theta) = -\sin(-\theta) = \sin\theta \end{array} \right.$$

Corioliskraft: $\vec{F}_C = 2m \, \vec{v} \times \vec{\omega}$

a) Süd → Nord: $\vec{v} = -v\vec{e}_x{}'$

$$\rightarrow \vec{F}_C = 2mv\omega \sin\theta \, \vec{e}_y{}'$$

nach Osten gerichtet

b) Ost → West: $\vec{v} = -v\vec{e}_y{}'$

$$F_C = -2mv \, \vec{e}_y{}' \times \left[-\omega\cos\theta \, \vec{e}_x{}' + \omega \sin\theta \, \vec{e}_z{}' \right]$$

$$= -2mv\omega \left(\underbrace{\vec{e}_x{}' \times \vec{e}_y{}'}_{= \vec{e}_z{}'} \cdot \cos\theta + \underbrace{\vec{e}_y{}' \times \vec{e}_z{}'}_{= \vec{e}_x{}'} \sin\theta \right)$$

$$= -2mv\omega \left(\cos\theta \, \vec{e}_z{}' + \sin\theta \, \vec{e}_x{}' \right)$$

nach Süden gerichtet

P18

To the fixed observer the unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$ actually change with time. Hence such an observer would compute the time derivative as

$$\frac{d\mathbf{A}}{dt} = \frac{dA_1}{dt}\mathbf{i} + \frac{dA_2}{dt}\mathbf{j} + \frac{dA_3}{dt}\mathbf{k} + A_1\frac{d\mathbf{i}}{dt} + A_2\frac{d\mathbf{j}}{dt} + A_3\frac{d\mathbf{k}}{dt} \tag{1}$$

i.e.,

$$\left.\frac{d\mathbf{A}}{dt}\right|_F = \left.\frac{d\mathbf{A}}{dt}\right|_M + A_1\frac{d\mathbf{i}}{dt} + A_2\frac{d\mathbf{j}}{dt} + A_3\frac{d\mathbf{k}}{dt} \tag{2}$$

Since $\mathbf{i}$ is a unit vector, $d\mathbf{i}/dt$ is perpendicular to $\mathbf{i}$ and must therefore lie in the plane of $\mathbf{j}$ and $\mathbf{k}$. Then

$$d\mathbf{i}/dt = \alpha_1\mathbf{j} + \alpha_2\mathbf{k} \tag{3}$$

Similarly,

$$d\mathbf{j}/dt = \alpha_3\mathbf{k} + \alpha_4\mathbf{i} \tag{4}$$

$$d\mathbf{k}/dt = \alpha_5\mathbf{i} + \alpha_6\mathbf{j} \tag{5}$$

From $\mathbf{i}\cdot\mathbf{j} = 0$, differentiation yields $\mathbf{i}\cdot\frac{d\mathbf{j}}{dt} + \frac{d\mathbf{i}}{dt}\cdot\mathbf{j} = 0$. But $\mathbf{i}\cdot\frac{d\mathbf{j}}{dt} = \alpha_4$ from (4) and $\frac{d\mathbf{i}}{dt}\cdot\mathbf{j} = \alpha_1$ from (3). Thus $\alpha_4 = -\alpha_1$.

Similarly from $\mathbf{i}\cdot\mathbf{k} = 0$, $\mathbf{i}\cdot\frac{d\mathbf{k}}{dt} + \frac{d\mathbf{i}}{dt}\cdot\mathbf{k} = 0$ and $\alpha_5 = -\alpha_2$; from $\mathbf{j}\cdot\mathbf{k} = 0$, $\mathbf{j}\cdot\frac{d\mathbf{k}}{dt} + \frac{d\mathbf{j}}{dt}\cdot\mathbf{k} = 0$ and $\alpha_6 = -\alpha_3$. Then

$$d\mathbf{i}/dt = \alpha_1\mathbf{j} + \alpha_2\mathbf{k}, \quad d\mathbf{j}/dt = \alpha_3\mathbf{k} - \alpha_1\mathbf{i}, \quad d\mathbf{k}/dt = -\alpha_2\mathbf{i} - \alpha_3\mathbf{j}$$

It follows that

$$A_1\frac{d\mathbf{i}}{dt} + A_2\frac{d\mathbf{j}}{dt} + A_3\frac{d\mathbf{k}}{dt} = (-\alpha_1A_2 - \alpha_2A_3)\mathbf{i} + (\alpha_1A_1 - \alpha_3A_3)\mathbf{j} + (\alpha_2A_1 + \alpha_3A_2)\mathbf{k} \tag{6}$$

which can be written as

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \alpha_3 & -\alpha_2 & \alpha_1 \\ A_1 & A_2 & A_3 \end{vmatrix}$$

Then if we choose $\alpha_3 = \omega_1$, $-\alpha_2 = \omega_2$, $\alpha_1 = \omega_3$ this determinant becomes

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_1 & \omega_2 & \omega_3 \\ A_1 & A_2 & A_3 \end{vmatrix} = \boldsymbol{\omega}\times\mathbf{A}$$

where $\boldsymbol{\omega} = \omega_1\mathbf{i} + \omega_2\mathbf{j} + \omega_3\mathbf{k}$.

From (2) and (6) we find, as required,

$$\left.\frac{d\mathbf{A}}{dt}\right|_F = \left.\frac{d\mathbf{A}}{dt}\right|_M + \boldsymbol{\omega}\times\mathbf{A}$$

The vector quantity $\boldsymbol{\omega}$ is the *angular velocity* of the moving system relative to the fixed system.