(a)

$$\boldsymbol{\Theta}_{S_1}=\frac{m_1}{12}\begin{bmatrix} b^2 & 0 & 0\\ 0 & a^2 & 0\\ 0 & 0 & a^2+b^2\end{bmatrix},\qquad \boldsymbol{\Theta}_{S_2}=\frac{m_2}{12}\begin{bmatrix} b^2+c^2 & 0 & 0\\ 0 & c^2 & 0\\ 0 & 0 & b^2\end{bmatrix},$$

$$\overrightarrow{AS_1}=\begin{bmatrix} a/2\\ b/2\\ 0\end{bmatrix},\qquad \overrightarrow{AS_2}=\begin{bmatrix} 0\\ b/2\\ c/2\end{bmatrix},$$

$$\boldsymbol{\vartheta}(\overrightarrow{AS_1})=\begin{bmatrix} b^2/4 & -ab/4 & 0\\ -ab/4 & a^2/4 & 0\\ 0 & 0 & (a^2+b^2)/4\end{bmatrix},\qquad \boldsymbol{\vartheta}(\overrightarrow{AS_2})=\begin{bmatrix} (b^2+c^2)/4 & 0 & 0\\ 0 & c^2/4 & -bc/4\\ 0 & -bc/4 & b^2/4\end{bmatrix},$$

$$\boldsymbol{\Theta}_A=\boldsymbol{\Theta}_{S_1}+m_1\boldsymbol{\vartheta}(\overrightarrow{AS_1})+\boldsymbol{\Theta}_{S_2}+m_2\boldsymbol{\vartheta}(\overrightarrow{AS_2}),$$

$$\boldsymbol{\Theta}_A=m_1\begin{bmatrix} b^2/3 & -ab/4 & 0\\ -ab/4 & a^2/3 & 0\\ 0 & 0 & (a^2+b^2)/3\end{bmatrix}+m_2\begin{bmatrix} (b^2+c^2)/3 & 0 & 0\\ 0 & c^2/3 & -bc/4\\ 0 & -bc/4 & b^2/3\end{bmatrix}.$$

(b)

Spezialfall: $m_1=m_2=m, \qquad a=b=c=L$

$$\boldsymbol{\Theta}_A=\frac{mL^2}{12}\begin{bmatrix} 12 & -3 & 0\\ -3 & 8 & -3\\ 0 & -3 & 12\end{bmatrix}.$$

Charakteristische Gleichung:

$$\det\begin{bmatrix} 12-\lambda & -3 & 0\\ -3 & 8-\lambda & -3\\ 0 & -3 & 12-\lambda\end{bmatrix}=0,\qquad (12-\lambda)(\lambda^2-20\lambda+78)=0,$$

$$\lambda_I=12,\qquad \lambda_{II}=10-\sqrt{22}=5{,}31,\qquad \lambda_{III}=10+\sqrt{22}=14{,}69.$$

Hauptträgheitsmomente:

$$\Theta_I=mL^2,\qquad \Theta_{II}=0{,}44mL^2,\qquad \Theta_{III}=1{,}22mL^2.$$

Trägheitshauptachsenrichtungen:

$$\begin{bmatrix} 12-\lambda_H & -3 & 0\\ -3 & 8-\lambda_H & -3\\ 0 & -3 & 12-\lambda_H\end{bmatrix}\begin{bmatrix} a_1\\ a_2\\ a_3\end{bmatrix}=\begin{bmatrix} 0\\ 0\\ 0\end{bmatrix},$$

$$\lambda_H=\lambda_I\qquad\rightarrow\qquad a_2=0,\ \ a_1=1,\ \ a_3=-1,\qquad \vec{e}_I=\frac{1}{\sqrt{2}}\begin{bmatrix} 1\\ 0\\ -1\end{bmatrix},$$

$$\lambda_H=\lambda_{II}\qquad\rightarrow\qquad a_1=1,\ \ a_2=\frac{12-\lambda_{II}}{3},\ \ a_3=1,\qquad \vec{e}_{II}=\begin{bmatrix} 0{,}3787\\ 0{,}8445\\ 0{,}3787\end{bmatrix},$$

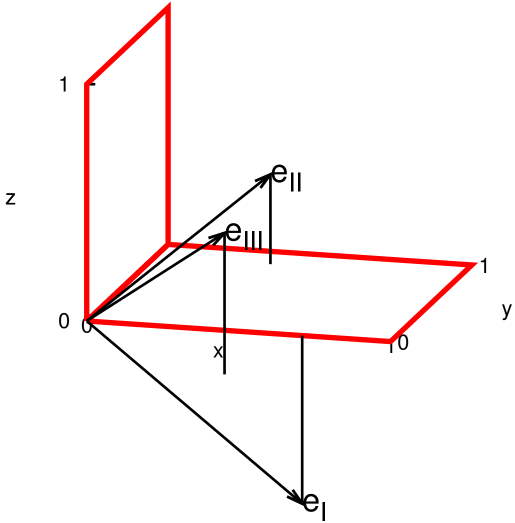
$$\vec{e}_{III}=\vec{e}_I\times\vec{e}_{II}=\begin{bmatrix} 0{,}5972\\ -0{,}5355\\ 0{,}5972\end{bmatrix}.$$

$$\boldsymbol{\Theta}_A=\frac{mL^2}{12}\begin{bmatrix} 12 & 0 & 0\\ 0 & 5{,}31 & 0\\ 0 & 0 & 14{,}69\end{bmatrix}_{HAS}$$

Trägheitsmoment um die Achse AS_1:

$$\vec{e}=\frac{1}{\sqrt{2}}\begin{bmatrix} 1\\ 1\\ 0\end{bmatrix},\qquad \Theta_{A(\vec{e})}=\vec{e}\cdot(\boldsymbol{\Theta}_A\vec{e})=\frac{1}{2}\begin{bmatrix} 1\\ 1\\ 0\end{bmatrix}\cdot\left(\frac{mL^2}{12}\begin{bmatrix} 12 & -3 & 0\\ -3 & 8 & -3\\ 0 & -3 & 12\end{bmatrix}\begin{bmatrix} 1\\ 1\\ 0\end{bmatrix}\right)$$

$$\Theta_{A(\vec{e})}=\frac{7mL^2}{12}.$$



1
z
0
0
x
e_{II}
e_{III}
e_{I}
1
y
0