P19

(a) The apparent velocity at any time $t$ is

$$d\mathbf{r}/dt \ = \ 2t\mathbf{i} - 6\mathbf{j} + 12t^2\mathbf{k}$$

At time $t = 1$ this is $2\mathbf{i} - 6\mathbf{j} + 12\mathbf{k}$.

The true velocity at any time $t$ is

$$d\mathbf{r}/dt + \boldsymbol{\omega}\times\mathbf{r} \ = \ (2t\mathbf{i} - 6\mathbf{j} + 12t^2\mathbf{k}) + [2t\mathbf{i} - t^2\mathbf{j} + (2t+4)\mathbf{k}] \times [(t^2+1)\mathbf{i} - 6t\mathbf{j} + 4t^3\mathbf{k}]$$

At time $t = 1$ this is

$$2\mathbf{i} - 6\mathbf{j} + 12\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 6 \\ 2 & -6 & 4 \end{vmatrix} \ = \ 34\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$$

(b) The apparent acceleration at any time $t$ is

$$\frac{d^2\mathbf{r}}{dt^2} = \frac{d}{dt}\left(\frac{d\mathbf{r}}{dt}\right) = \frac{d}{dt}(2t\mathbf{i} - 6\mathbf{j} + 12t^2\mathbf{k}) = 2\mathbf{i} + 24t\mathbf{k}$$

At time $t = 1$ this is $2\mathbf{i} + 24\mathbf{k}$.

The true acceleration at any time $t$ is

$$\frac{d^2\mathbf{r}}{dt^2} + 2\boldsymbol{\omega}\times\frac{d\mathbf{r}}{dt} + \frac{d\boldsymbol{\omega}}{dt}\times\mathbf{r} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r})$$

At time $t = 1$ this equals

$$\begin{aligned} & 2\mathbf{i} + 24\mathbf{k} + (4\mathbf{i} - 2\mathbf{j} + 12\mathbf{k}) \times (2\mathbf{i} - 6\mathbf{j} + 12\mathbf{k}) \\ & \quad + (2\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) \times (2\mathbf{i} - 6\mathbf{j} + 4\mathbf{k}) \\ & \quad\quad + (2\mathbf{i} - \mathbf{j} + 6\mathbf{k}) \times \{(2\mathbf{i} - \mathbf{j} + 6\mathbf{k}) \times (2\mathbf{i} - 6\mathbf{j} + 4\mathbf{k})\} \\ = \ & 2\mathbf{i} + 24\mathbf{k} + (48\mathbf{i} - 24\mathbf{j} - 20\mathbf{k}) + (4\mathbf{i} - 4\mathbf{j} - 8\mathbf{k}) + (-14\mathbf{i} + 212\mathbf{j} + 40\mathbf{k}) \\ = \ & 40\mathbf{i} + 184\mathbf{j} + 36\mathbf{k} \end{aligned}$$

(c) From Problem (a) we have,

$$\text{Coriolis acceleration} = 2\boldsymbol{\omega}\times d\mathbf{r}/dt = (4\mathbf{i} - 2\mathbf{j} + 12\mathbf{k}) \times (2\mathbf{i} - 6\mathbf{j} + 12\mathbf{k}) = 48\mathbf{i} - 24\mathbf{j} - 20\mathbf{k}$$

From Problem (a) we have,

$$\text{Centripetal acceleration} = \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) = (2\mathbf{i} - \mathbf{j} + 6\mathbf{k}) \times (32\mathbf{i} + 4\mathbf{j} - 10\mathbf{k}) = -14\mathbf{i} + 212\mathbf{j} + 40\mathbf{k}$$

$$\text{Magnitude of Coriolis acceleration} = \sqrt{(48)^2 + (-24)^2 + (-20)^2} = 4\sqrt{205}$$

$$\text{Magnitude of centripetal acceleration} = \sqrt{(-14)^2 + (212)^2 + (40)^2} = 2\sqrt{11{,}685}$$

P20

Choose the $xyz$ coordinate system of Fig. 1
Suppose that the origin $O$ is the equilibrium position of the bob $B$, $A$ is the point of suspension and the length of string $AB$ is $l$. If the tension in the string is $\mathbf{T}$, then we have

$$\begin{aligned}\mathbf{T} &= (\mathbf{T}\cdot\mathbf{i})\mathbf{i} + (\mathbf{T}\cdot\mathbf{j})\mathbf{j} + (\mathbf{T}\cdot\mathbf{k})\mathbf{k} \\ &= T\cos\alpha\,\mathbf{i} + T\cos\beta\,\mathbf{j} + T\cos\gamma\,\mathbf{k} \\ &= -T\left(\frac{x}{l}\right)\mathbf{i} - T\left(\frac{y}{l}\right)\mathbf{j} + T\left(\frac{l-z}{l}\right)\mathbf{k} \qquad (1)\end{aligned}$$

Since the net force acting on $B$ is $\mathbf{T} + m\mathbf{g}$, the equation of motion of $B$ is given by [see Problem 6.14]

$$m\frac{d^2\mathbf{r}}{dt^2} = \mathbf{T} + m\mathbf{g} - 2m(\boldsymbol{\omega}\times\mathbf{v}) - m\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) \qquad (2)$$

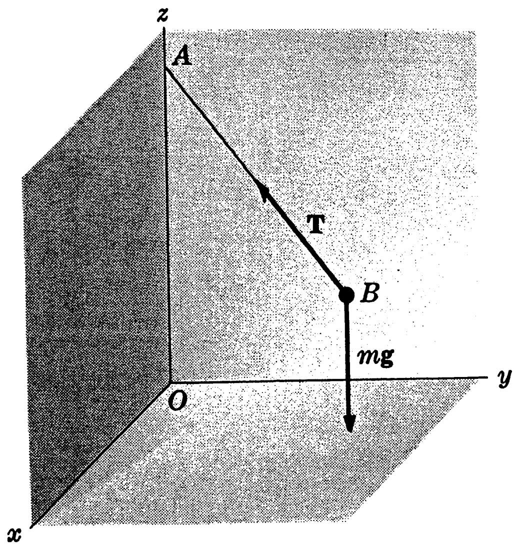


Fig 1

If we neglect the last term in (2), put $\mathbf{g} = -g\mathbf{k}$ and use (1), then (2) can be written in component form as

$$m\ddot{x} = -T(x/l) + 2m\omega\dot{y}\cos\lambda \qquad (3)$$

$$m\ddot{y} = -T(y/l) - 2m\omega(\dot{x}\cos\lambda + \dot{z}\sin\lambda) \qquad (4)$$

$$m\ddot{z} = T(l-z)/l - mg + 2m\omega\dot{y}\sin\lambda \qquad (5)$$