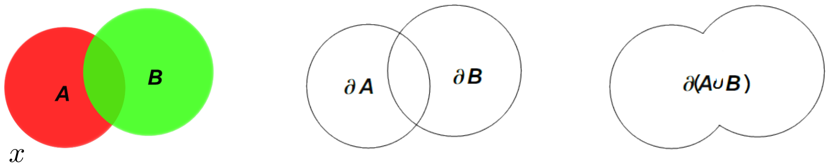
Besprechung in Mathematik für Physiker II zur 1. Klausur

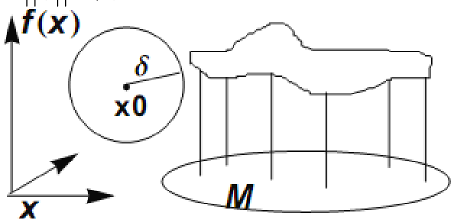
Nr 2



c) c)$\overline{A \cup B} = \overline{A} \cup \overline{B}$
Bew: „$\subset$“: Sei $x \in \overline{A \cup B}$
Es ex. $(x_n) \subset A \cup B, x_n \to x$
Es ex. Teilf. $(x_{\varphi(n)}) \subset (x_n), (x_{\varphi(n)}) \subset A$ oder $(x_{\varphi(n)}) \subset (x_n), (x_{\varphi(n)}) \subset B$.
Im ersten Fall: $x_{\varphi(n)} \to x, (x_{\varphi(n)}) \subset A$. Also $x \in \overline{A}$ zweiten Fall analog: $x \in \overline{B}$.
Also $x \in \overline{A} \cup \overline{B}$
„$\supset$“: Sei $x \in \overline{A} \cup \overline{B}$. Falls $x \in \overline{A}$, so ex, Folge $(x_n) \subset A$, $x_n \to x$.
Dann auch $(x_n) \subset A \cup B$, also $x \in \overline{A \cup B}$. Analog $B \subset \overline{A \cup B}$

d) $\partial(A \cup B) = \overline{A \cup B} \setminus (A \mathring{\cup} B) = (\overline{A} \cup \overline{B}) \setminus (A \mathring{\cup} B) = [\overline{A} \setminus (A \mathring{\cup} B)] \cup [\overline{B} \setminus (A \mathring{\cup} B)]$
$\partial A \cup \partial B = [\overline{A} \setminus \mathring{A}] \cup [\overline{B} \setminus \mathring{B}]$
Wegen $\mathring{A} \subset (A \mathring{\cup} B), \mathring{B} \subset (A \mathring{\cup} B) \Rightarrow \partial A \cup \partial B \supset \partial(A \cup B)$

b) x_0 isol. P
Also ex.$\delta > 0 : B_{||.||}(x_0, \delta) \cap M = \{x_0\}$.
Sei $\varepsilon > 0$. Für $x \in M$, $||x - x_0|| < \delta$ gilt $x = x_0, ||f(x) - f(x_0)|| = ||0|| < \varepsilon$



Nr 5

b) $A = \begin{pmatrix} 1 & 3 & 0 \\ 2 & -1 & 3 \\ -2 & 4 & 1 \end{pmatrix}$. $\hat{A} = (\mathbb{R}^3, ||.||_2) \to (\mathbb{R}^3, ||.||_\infty)$
$||A|| = \sup\limits_{||x||_2 \leq 1} ||Ax||_\infty$
Falls $x = (x_1, x_2, x_3)$, $||x||_2 \leq 1 \Rightarrow |x_i| < 1$, $i = 1, 2, 3$.
Also $||Ax||_\infty = \max\{|x_1 + 3x_2|, |2x_1 - x_2 + 3x_3|, |-2x_1 + 4x_2 + x_3|\} \leq \max\{1 + 3, 2 + 1 + 3, 2 + 4 + 1\} = 7$

c) $$||Ax||_\infty = \max\{ \underbrace{|x_1 + 3x_2|}_{\leq ||(1,3,0)||_2 - \underbrace{||x||_2}_{\leq 1}}, \underbrace{|2x_1 - x_2 + 3x_3|}_{\leq ||(2,1,3)||_2 - \underbrace{||x||_2}_{\leq 1}}, \underbrace{|-2x_1 + 4x_2 + x_3|}_{\leq \underbrace{||(-2,4,1)||_2}_{\sqrt{21}} - \underbrace{||x||_2}_{\leq 1}} \} \leq \sqrt{21}$$

Nr 3

a) $J_f(r\varphi) = \begin{pmatrix} 1 + \cos(\varphi) & -r\sin(\varphi) \\ \sin(\varphi) & r\cos(\varphi) \\ 2r & 0 \end{pmatrix}$

$J_g(x, y, z) = \begin{pmatrix} y, x, 0 \\ 1, 0, -1 \end{pmatrix}$

$$J_{g \circ f}(r, \varphi) = J_g(f(r, \varphi)) \cdot J_f(r, \varphi) = \begin{pmatrix} rs & r + rc & 0 \\ 1 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 + c & -rs \\ s & rc \\ 2r & 0 \end{pmatrix}$$

$$= \begin{pmatrix} rs(1 + c) + r(1 + c)s & -rs^2 + r^2c + rc^2 \\ 1 + c - 2r & -rc \end{pmatrix} = \begin{pmatrix} 2rs(1 + c) & r(c^2 - s^2) + r^2c \\ 1 + c - 2r & -rs \end{pmatrix}$$

mit $s = \sin(\varphi)$, $c = cos\varphi$

Test: $(g \circ f)(r, \varphi) = (r(1 + \cos(\varphi))r\sin(\varphi), r(1 + \cos(\varphi) - r^2) = (r^2(1 + \cos(\varphi)\sin(\varphi), r(1 + \cos(\varphi) - r^2)$

$$J_{g \circ f} = \begin{pmatrix} 2r(1 + c)s & r^2(c^2 - s^2) + r^2c \\ -2r + 1 + c & -rs \end{pmatrix}$$