

7 Präsenzblatt vom 29.11.12

7.1 Besprechung Blatt 5

$$\begin{aligned} \mu, \nu \in \{0, 1, 2, 3\} = SK &\Rightarrow g_{\mu, \nu} = g^{\mu\nu} \\ \alpha, \beta, \gamma \in \{+, -, 1, 2\} = LK &\Rightarrow \tilde{g}^{\alpha\beta}; \tilde{g}_{\alpha\beta} \\ x^\alpha y_\alpha = x^\mu y_\mu = x^0 y^0 - x^1 y^1 - x^2 y^2 - x^3 y^3 \\ &= x^\alpha \tilde{g}_{\alpha\beta} y^\beta \\ a^\pm = a^0 \pm a^3 &\Leftrightarrow \begin{cases} a^0 = \frac{a^+ + a^-}{2} \\ a^3 = \frac{a^+ - a^-}{2} \end{cases} \\ \tilde{g}_{\alpha\beta} = (A^T)^{-1} \tilde{g}_{\mu\nu} A^{-1} \\ \Rightarrow x^\alpha \tilde{g}_{\alpha\beta} y^\beta = \frac{1}{2} x^+ y^- + \frac{1}{2} x^- y^+ - x^1 y^1 - x^2 y^2 \\ (\tilde{g}^{\alpha\beta})^{-1} = \tilde{g}_{\alpha\beta} = \left(\begin{array}{c|cccc} & + & - & 1 & 2 \\ \hline + & 0 & 0.5 & 0 & 0 \\ - & 0.6 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 2 & 0 & 0 & 0 & -1 \end{array} \right) \\ g^{\gamma\alpha} x_\alpha = g^{\gamma\alpha} g_{\alpha\beta} x^\beta = x^\gamma = (g^{\gamma\alpha} g_{\alpha\beta}) x^\beta \end{aligned}$$

7.2 Präsenzaufgabe 7

$$\begin{aligned} \text{a) } \gamma^5 &= i\gamma^0\gamma^1\gamma^2\gamma^3 \\ \{\gamma^\mu, \gamma^\nu\} &= 2g^{\mu\nu} \cdot 1_4 \\ g^{\mu\nu} &= \left(\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) \Bigg|_{\mu\nu} \\ \Rightarrow \{\gamma^\mu, \gamma^\nu\} &= 0 \forall \mu \neq \nu \text{ mit } \mu, \nu \in \{0, 1, 2, 3\} \\ \gamma^0\gamma^5 &= \gamma^0 i\gamma^0\gamma^1\gamma^2\gamma^3 = -i\gamma^0\gamma^1\gamma^0\gamma^2\gamma^3 = -i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^0 = \gamma^5\gamma^0 \\ \Rightarrow \{\gamma^\nu, \gamma^5\} &= 0 \\ \text{b) } \gamma^\lambda\gamma_\lambda &= \sum_{\alpha\lambda} \gamma^\lambda g_{\lambda\alpha} \gamma^\alpha = g_{\lambda\alpha} \gamma^\lambda \gamma^\alpha = g_{\lambda\alpha} (2g^{\lambda\alpha} 1_4 - \gamma^\alpha \gamma^\lambda) \\ &= 2g_{\lambda\alpha} g^{\lambda\alpha} 1_4 - \gamma^\alpha g_{\alpha\lambda} \gamma^\lambda = 2 \cdot 1_4 \sum_{\alpha} \underbrace{g_{\alpha\lambda} g^{\lambda\alpha}}_{g_\alpha^\alpha} \\ \Leftrightarrow \gamma^\lambda\gamma_\lambda &= 1_4 \sum_{\alpha} g_\alpha^\alpha = 4 \cdot 1_4 \\ &\quad \underbrace{\alpha}_{Sp(14)} \end{aligned}$$