

1 Besprechung zu Blatt1 zu Analysis 3

Aufgabe 3

$$k = V$$

$$Kbel : m(t) := \int_K \rho(t, x) dx$$

$$K \subseteq \mathbb{R}^3, \partial K \text{-} 2\text{-dim. } C^2 \text{ Umft. } \dot{m}(t) = - \int_{\partial K} \rho(t, x) \langle v(t, x), \nu \rangle dx$$

$\forall K \subseteq \mathbb{R}^3, \partial K \text{ 2-dim } C^2 \text{ Umft gilt:}$

$$- \int_{\partial K} \rho(t, *) \langle v(t, *), \nu \rangle ds = \dot{m}(t) = \frac{d}{dt} \int_K \rho(t, x) dx = - \int_{\partial K} \langle \rho(t, *) v(t, *), \nu \rangle ds =$$

$$- \int_K \operatorname{div}(\rho v)(t, x) dx = \int_K \frac{\partial}{\partial t} \rho(t, x) dx$$

$$\Rightarrow \int_K \left(\frac{\partial}{\partial t} \rho(t, x) + \operatorname{div}(\rho v)(t, x) \right) dx = 0$$

$$Kbel., \frac{\partial}{\partial t} \rho(t, *) + \operatorname{div}(\rho v)(t, *) \text{ stetig} \Rightarrow \frac{\partial}{\partial t} \rho(t, x) + \operatorname{div}(\rho v)(t, x) = 0$$

Aufgabe 4

OE. $\operatorname{div}(f) > 0$

$\gamma : I \rightarrow \mathbb{R}^2$ Lösung von $\dot{x} = f(x)$, periodisch, nicht konstant, dann umschließt das $\operatorname{Bild}(\gamma)$

eine kompakte Menge K mit $\overset{\circ}{K} \neq \emptyset$

$$\operatorname{vol}(K) \geq \operatorname{vol}(\overset{\circ}{K}) > 0$$

$$\int_K \operatorname{div}(f) = \int_{\partial K} \langle f, \nu \rangle dS$$

$$\gamma C^1 \Rightarrow \partial K = \operatorname{Bild}(\gamma) C^1, \exists \nu$$

$$\operatorname{div}(f) \text{ stetig, } K \text{ kompakt} \Rightarrow c := \min_{x \in K} \operatorname{div}(f(x)) > 0$$

$$\int_K \operatorname{div}(f) \geq \int_K c = c \cdot \operatorname{vol}(K) > 0 \text{ (kleiner für max)}$$

$$\int_{\partial K} \langle f, \nu \rangle dS \text{ Oberflächenintegral der Dim 1, Param: } \gamma : I \rightarrow \mathbb{R}^2$$

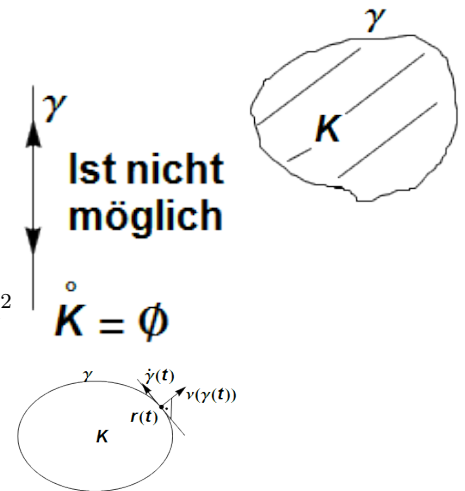
$$\dots = \int_I \langle f(\gamma(t)), \nu(\gamma(t)) \rangle \sqrt{\det(g_{ij})} dt$$

$$g_{11} = (g_{ij})_{i,j=1} = \langle \dot{\gamma}(t), \dot{\gamma}(t) \rangle = \|\dot{\gamma}(t)\|_2^2$$

$$\text{Beh. } \forall t \in I \text{ ist } \langle f(\gamma(t)), \nu(\gamma(t)) \rangle > 0$$

$$\gamma(t) \in \partial K, T_{\gamma(t)} \partial K = \operatorname{span}\{\dot{\gamma}(t)\}$$

$$\left. \begin{array}{l} \nu(\gamma(t)) \in (T_{\gamma(t)} \partial K)^\perp = \dot{\gamma}(t)^\perp \\ f(\gamma(t)) = \dot{\gamma}(t) \end{array} \right\} \Rightarrow \nu(\gamma(t)) \perp f(\gamma(t)) \Rightarrow \langle \nu(\gamma(t)), f(\gamma(t)) \rangle = 0$$



Aufgabe 5

a) $\alpha, \beta, \gamma, \delta > 0$

$$\dot{x} = \alpha x - \beta xy$$

$$\dot{y} = -\gamma y + \delta xy$$

aus der b) erhalten wir: Gleichgewichte $0, (1, 1)$ mit

$$J_f(0) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, J_f(1, 1) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Taylorpolynom 1. Grades für x, y nahe 0 von

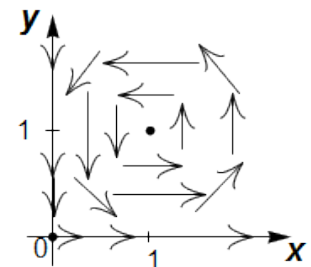
$$f(x, y) \approx f(0) + J_f(0)(x, y) = J_f(0)(x, y) = (x, -y)$$

$y_0 = 0 \Rightarrow 0$ Lösung von *

$$\dot{x} = x \Rightarrow x(t) = ce^t$$

$$x_0 = 0 \Rightarrow x = 0$$

$$\dot{y} = -y \Rightarrow y(t) = ce^{-t}$$



$$(x, y) \text{ nahe } (1, 1)$$

$$f(x, y) \approx f(1, 1) + \underbrace{J_f(1, 1)((x, y) - (1, 1))}_{\text{Drehmatrix mit Winkel } \pi/2} = (-y, x)$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \cos(\pi/2) & -\sin(\pi/2) \\ \sin(\pi/2) & \cos(\pi/2) \end{pmatrix}$$

b) Gleichgewichte: $(x, y) \in (0, \infty)^2$ mit $f(x, y) = 0$

$$f: \mathbb{R}^2 \rightarrow \mathbb{R}^2, f(x, y) = \begin{pmatrix} \alpha x - \beta xy \\ -\gamma y + \delta xy \end{pmatrix} = \begin{pmatrix} x(\alpha - \beta y) \\ y(-\gamma + \delta x) \end{pmatrix}$$

$$f(x, y) = 0 \Leftrightarrow (x, y) = 0 \text{ oder } [\alpha - \beta y = 0, -\gamma + \delta x = 0] \Leftrightarrow (x, y) = 0 \text{ oder } (x, y) = \left(\frac{\gamma}{\delta}, \frac{\alpha}{\beta}\right)$$

$$J_f(x, y) = \begin{pmatrix} \alpha - \beta y & -\beta x \\ \delta y & -\gamma + \delta x \end{pmatrix}$$

$$J_f(0) = \begin{pmatrix} \alpha & 0 \\ 0 & \gamma \end{pmatrix}, EW: \alpha, -\gamma$$

$$J_f\left(\frac{\gamma}{\delta}, \frac{\alpha}{\beta}\right) = \begin{pmatrix} 0 & -\frac{\beta\gamma}{\delta} \\ \frac{\delta\alpha}{\beta} & 0 \end{pmatrix} EW: \pm\sqrt{\alpha\gamma}$$

$$\text{charakteristisches Polynom: } \lambda^2 - \left(\frac{\beta\gamma}{\delta}\right)\frac{\delta\alpha}{\beta} = \lambda^2 + \gamma\alpha$$