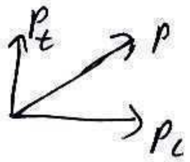
higher hadron physics Ex/Ho 4 Julian Bergmann

No 1

c) $E^2=p^2+m^2 \Rightarrow E=\sqrt{p^2+m^2}$



$\vartheta = 2\arctan(e^{-\eta})$

$\tan(\vartheta) = \frac{p_t}{p_l} = \frac{\sqrt{p^2-p_l^2}}{p_l} \Rightarrow p_l = \tan^{-1}(\vartheta)\sqrt{p^2-p_l^2}$

$\Rightarrow p_l^2 = \tan^{-2}(\vartheta)\sqrt{p^2-p_l^2}^2 = \frac{p^2-p_l^2}{\tan^2(\vartheta)}$

$\Rightarrow p_l^2\left(1+\frac{1}{\tan^2(\vartheta)}\right) = \frac{p^2}{\tan^2(\vartheta)} \Leftrightarrow p_l^2 = p^2\cdot\left(\tan^2(\vartheta)\left(1+\frac{1}{\tan^2(\vartheta)}\right)\right)^{-1}$

$\Rightarrow p_l^2 = p^2\left(\tan^2(\vartheta)+1\right)^{-1} \Rightarrow p_l = \frac{p}{\sqrt{\tan^2(\vartheta)+1}}$

	E (GeV)	p (GeV/c)	y	η	p_l (GeV/c)
p	2,23	2	0,835	1	1,523
K	2,06	2	1,704	2	1,928
π	2,0048	2	~~4,952~~ 3,34	5	1,9998

see back ⇒

d) $\tan\left(\frac{x}{2}\right) = \frac{\tan(x)}{1+\sqrt{1+\tan^2(x)}}$

$\eta = -\ln\left(\tan\left(\frac{\vartheta}{2}\right)\right) = -\ln\left(\frac{\frac{\sqrt{p^2-p_l^2}}{p_l}}{1+\sqrt{1+\left(\frac{\sqrt{p^2-p_l^2}}{p_l}\right)^2}}\right)$

$= -\ln\left(\frac{\sqrt{p^2-p_l^2}}{p_l}\cdot\left(1+\frac{1}{p_l}\sqrt{p_l^2+p^2-p_l^2}\right)^{-1}\right) = -\ln\left(\frac{\sqrt{p^2-p_l^2}}{p_l}\left(\frac{p_l+p}{p_l}\right)^{-1}\right)$

$= -\ln\left(\frac{\sqrt{p^2-p_l^2}}{p_l+p}\right) = -\ln\left(\frac{\sqrt{p-p_l}\sqrt{p+p_l}}{\sqrt{p+p_l}\sqrt{p+p_l}}\right) = -\ln\left(\frac{\sqrt{p-p_l}}{\sqrt{p+p_l}}\right)$

$= \ln\left(\frac{\sqrt{p+p_l}}{\sqrt{p-p_l}}\right) = \frac{1}{2}\ln\left(\frac{p+p_l}{p-p_l}\right)$

$E\approx|p| \Rightarrow y = \frac{1}{2}\ln\left(\frac{E+p_l}{E-p_l}\right) \approx \frac{1}{2}\ln\left(\frac{p+p_l}{p-p_l}\right) = \eta$

	E (GeV)	p ($\frac{GeV}{c}$)	y	η	p_t ($\frac{GeV}{c}$)
p	2,23	2	1,311	2	1,928
K	2,06	2	2,105	5	1,9998
π	2,0043	2	0,995	1	1,523
p	2,23	2	1,455	5	1,9998
K	2,06	2	0,949	1	1,523
π	2,0043	2	1,968	2	1,928

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Homework 1

$$|\eta\rangle = X_\eta \frac{1}{\sqrt{2}}|u\bar{u}+d\bar{d}\rangle + Y_\eta |s\bar{s}\rangle$$

$$|\eta'\rangle = X_{\eta'} \frac{1}{\sqrt{2}}|u\bar{u}+d\bar{d}\rangle + Y_{\eta'} |s\bar{s}\rangle$$

$$|\eta_8\rangle = \frac{1}{\sqrt{6}}|u\bar{u}+d\bar{d}-2s\bar{s}\rangle$$

$$|\eta_0\rangle = \frac{1}{\sqrt{3}}|u\bar{u}+d\bar{d}+s\bar{s}\rangle$$

$$|\eta\rangle = \cos(\theta_p)|\eta_8\rangle - \sin(\theta_p)|\eta_0\rangle$$

$$|\eta'\rangle = \sin(\theta_p)|\eta_8\rangle + \cos(\theta_p)|\eta_0\rangle$$

$$X_\eta \frac{1}{\sqrt{2}}|u\bar{u}+d\bar{d}\rangle + Y_\eta|s\bar{s}\rangle = \cos(\theta_p)|\eta_8\rangle - \sin(\theta_p)|\eta_0\rangle$$

$$= \cos(\theta_p)\cdot\frac{1}{\sqrt{6}}|u\bar{u}+d\bar{d}-2s\bar{s}\rangle - \sin(\theta_p)\cdot\frac{1}{\sqrt{3}}|u\bar{u}+d\bar{d}+s\bar{s}\rangle$$

$$= \underbrace{\left(\cos(\theta_p)\cdot\frac{1}{\sqrt{6}} - \sin(\theta_p)\cdot\frac{1}{\sqrt{3}}\right)}_{X_{\eta}\frac{1}{\sqrt{2}}}|u\bar{u}+d\bar{d}\rangle + \underbrace{\left(\frac{-2}{\sqrt{6}}\cos(\theta_p) - \frac{1}{\sqrt{3}}\sin(\theta_p)\right)}_{Y_\eta}|s\bar{s}\rangle$$

$$\Rightarrow Y_\eta = -\sqrt{\frac{2}{3}}\cos(\theta_p) - \frac{1}{\sqrt{3}}\sin(\theta_p)$$

$$X_\eta = \sqrt{\frac{1}{3}}\cos(\theta_p) - \sqrt{\frac{2}{3}}\sin(\theta_p)$$

$$X_{\eta'}\cdot\frac{1}{\sqrt{2}}|u\bar{u}+d\bar{d}\rangle + Y_{\eta'}|s\bar{s}\rangle = \sin(\theta_p)|\eta_8\rangle + \cos(\theta_p)|\eta_0\rangle$$

$$= \sin(\theta_p)\cdot\frac{1}{\sqrt{6}}|u\bar{u}+d\bar{d}-2s\bar{s}\rangle + \cos(\theta_p)\cdot\frac{1}{\sqrt{3}}|u\bar{u}+d\bar{d}+s\bar{s}\rangle$$

$$= \underbrace{\left(\sin(\theta_p)\cdot\frac{1}{\sqrt{6}} + \cos(\theta_p)\frac{1}{\sqrt{3}}\right)}_{X_{\eta'}\frac{1}{\sqrt{2}}}|u\bar{u}+d\bar{d}\rangle + \underbrace{\left(\frac{-2}{\sqrt{6}}\sin(\theta_p) + \frac{1}{\sqrt{3}}\cos(\theta_p)\right)}_{Y_{\eta'}}|s\bar{s}\rangle$$

$$\Rightarrow X_{\eta'} = \sqrt{\frac{2}{3}}\cos(\theta_p) + \sqrt{\frac{1}{3}}\sin(\theta_p) = -Y_\eta$$

$$Y_{\eta'} = -\sqrt{\frac{2}{3}}\sin(\theta_p) + \frac{1}{\sqrt{3}}\cos(\theta_p) = X_\eta$$

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No 1

0^{+-} : Spin 0 or 1.

Spin 0: $J=0$, $S=0$, $J=L\oplus S \Rightarrow L=0$, $P=(-1)^{L+1}=-1$ ↯

Spin 1: $J=0$, $S=1$, $J=L\oplus S \Rightarrow L=0$ or 1

$L=0$: $P=(-1)^{L+1}=-1$ ↯

$L=1$: $P=(-1)^{L+1}=1$, $C=(-1)^{L+S}=1$ ↯

1^{-+} : Spin 0: $J=1$, $S=0$, $J=L\oplus S \Rightarrow L=1$

$L=1$: $P=(-1)^{L+1}=+1$, $C=(-1)^{L+S}=-1$ ↯

Spin 1: $J=1$, $S=1$, $J=L\oplus S \Rightarrow L=0$ or 1 or 2

$L=0$: $P=(-1)^{L+1}=-1$, $C=(-1)^{L+S}=-1$ ↯

$L=1$: $P=(-1)^{L+1}=1$ ↯

$L=2$: $P=(-1)^{L+1}=-1$, $C=(-1)^{L+S}=-1$ ↯

2^{+-} : Spin 0: $J=2$, $S=0$, $J=L\oplus S \Rightarrow L=2$

$L=0$: $P=(-1)^{L+1}=-1$ ↯

Spin 1: $J=2$, $S=1$, $J=L\oplus S \Rightarrow L=1$ or 2 or 3

$L=1$: $P=(-1)^{L+1}=1$, $C=(-1)^{L+S}=1$ ↯

$L=2$: $P=(-1)^{L+1}=-1$ ↯

$L=3$: $P=(-1)^{L+1}=1$, $C=(-1)^{L+S}=1$ ↯

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No 2

a) $\rho \to \pi^+\pi^-$:
charge : $0 = 1 - 1$
parity : $-1 = (-1)\cdot 1 \cdot 1 \leftarrow (-1)^L$
ang. mom. : $\underset{J_\rho}{1} \oplus \underset{S_\rho}{1} = \underset{J_{\pi^+}}{0} + \underset{J_{\pi^-}}{0} + \underset{L}{0}$
Isospin : $1 = 1 \oplus 1$
I_z : $0 = 1 - 1$
G-Parity : $1 = (-1)(-1)$ ⇒ permitted

$\omega \to \pi^+\pi^-$:
Isospin : $1 = 1 \oplus 1$
G-Parity : $-1 \neq (-1)(-1)$ ⇒ not allowed
(rest similar to ρ)

b) $\rho \to \pi^0\pi^0$:
charge : $0 = 0 + 0$
parity : $-1 = (-1)(-1)(-1)^1$
ang. mom. : $1 \oplus 1 = 0 + 0 + 1$
C-Parity : $-1 \neq 1 \cdot 1$ ⇒ not allowed
CP-viol. : $1 = (-1)(-1)$
Isospin : $1 = 1 \oplus 1$
I_z : $0 = 0 + 0$
G-Parity : $1 = (-1)(-1)$

$\omega \to \pi^0\pi^0$:
Isospin : $0 = 1 \oplus 1$
G-Parity : $-1 \neq (-1)(-1)$ ⇒ not allowed
C-Parity : $-1 \neq 1 \cdot 1$

c) $\rho \to \eta\pi^0$:
charge : $0 = 0 + 0$
parity : $-1 = 1 \cdot (-1) \cdot 1$
ang. mom. : $1 \oplus 1 = 0 + 0 + 0$
C-Parity : $-1 \neq 1 \cdot 1$ ⇒ not allowed
CP-viol. : $1 \neq 1 \cdot (-1)$
Isospin : $1 = 1 \oplus 0$
I_z : $0 = 0 + 0$
G-Parity : $1 \neq (-1) \cdot 1$

$\omega \to \eta\pi^0$:
Isospin : $0 \neq 1 \oplus 0$ ⇒ not allowed
G-Parity : $(-1) = (-1) \cdot 1$
C-Parity : $-1 \neq 1 \cdot 1$
CP-viol. : $1 \neq 1 \cdot (-1)$

d) $\rho^+ \to \eta\pi^+$:
charge : $1 = 0 + 1$
parity : $-1 = (-1)(-1)(-1)^1$
ang. mom. : $1 \oplus 1 = 0 + 0 + 1$
Isospin : $1 \neq 0 \oplus 1$ ⇒ not allowed
I_z : $1 = 0 + 1$
G-parity : $1 \neq 1 \cdot (-1)$

c) $J/\psi \to \pi^0 \pi^0$:

charge : $0 = 0 + 0$
parity : $-1 = 1 \cdot 1 \cdot (-1)^1$
ang. mom. : $1 \oplus 1 = 0 + 0 + 1$
C-Parity : $-1 \neq (-1)(-1)$ $\Rightarrow$ not allowed
CP-viol. : $1 = (-1)(-1)$
Isospin : $0 = 1 \oplus 1$
I_z : $0 = 0 + 0$
G-Parity : $-1 \neq (-1)(-1)$

$J/\psi \to \pi^+ \pi^-$:

charge : $0 = 1 - 1$
parity : $-1 = (-1) \cdot 1 \cdot 1$
ang. mom. : $1 \oplus 1 = 0 + 0 + 0$
G-Parity : $-1 \neq (-1)(-1)$ $\Rightarrow$ not allowed
Isospin : $0 = 1 \oplus 1$
I_z : $0 = 1 - 1$