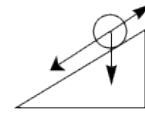


# Besprechung zu Blatt 7 zu Analysis 3

## Aufgabe 6



$$\ddot{x} = \underbrace{V'(x(t))}_{\text{Kraft}} + \underbrace{\dot{x}(t)}_{\text{Reibung}}$$

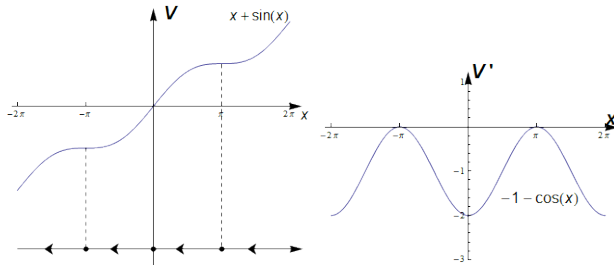
riesige Kraft, riesige Reibung:  $\ddot{x} = \underbrace{\mu}_{\rightarrow \infty} \underbrace{(V'(x(t)) + \dot{x}(t))}_{\rightarrow 0} \Rightarrow V'(x(t)) + \dot{x}(t) = 0 \Leftrightarrow$

$$\dot{x}(t) = -V'(x(t))$$

i)  $V : \mathbb{R} \rightarrow \mathbb{R}, V(x) = x + \sin(x)$

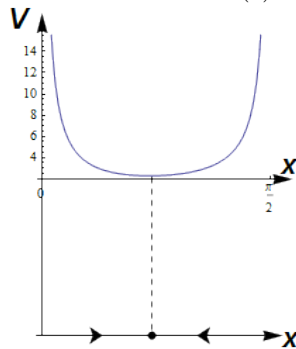
$$-V'(x) = -1 - \cos(x)$$

Gleichgewichte:  $\pi + 2\pi$

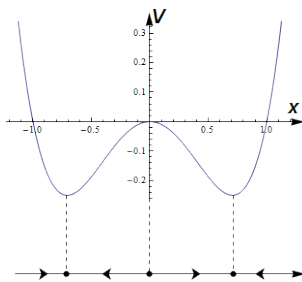


ii)  $V(x) = \frac{1}{x} + \tan(x)$

$$-V'(x) = \frac{1}{x^2} - \frac{1}{\cos^2(x)} = -\frac{x^2 - \cos^2(x)}{x^2 \cos^2(x)}$$



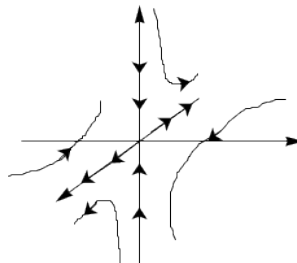
iii)



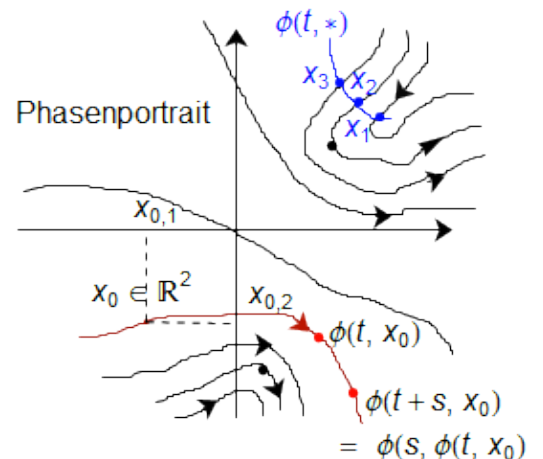
## Phasenportraits

Für  $A \in \mathbb{R}^{2 \times 2}$   
seien 1 EW,  $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$  EV zu 1

-2 EW,  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$  EV zu  $-2 \dot{x} = Ax$



## Phasenportrait



## Fluss

$$\phi(*, x_0)$$

$$\dot{\phi}(t, x_0) = f(\phi(t, x_0))$$

$$\phi(0, x_0) = x_0$$

$$\phi(s + t, x_0) = \phi(s, \phi(t, x_0))$$

$$x_0^1, x_0^2, x_0^3 \text{ nahe beieinander, } f \in C^2 \Rightarrow \phi(t, x_1), \phi(t, x_2), \phi(t, x_3) \text{ nahe beieinander } \forall t \in I_{x_0^1}$$

## Niveaulinien

$$E = \frac{\omega^2}{2} + (1 - \cos(\varphi))$$

Weitere Überlegungen machen aus dem Energieniveau fast das Phasenportrait (eventuell)

Unsere Überlegung mit Pendel:

