Numerik 1 Blatt 9 Julian Bergmann

Aufgabe 1

10

$X = \{-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\}$, $S_{x,3}$?

$f_1(x) := |x|^3 = \begin{cases} x^3 & x \geq 0 \\ -x^3 & x < 0 \end{cases}$

$f_1(x)|_{[-1,-\frac{1}{2}]} = -x^3 \in \mathbb{P}^3$, $f_1(x)|_{[-\frac{1}{2},0]} = -x^3 \in \mathbb{P}^3$, $f_1(x)|_{[0,\frac{1}{2}]} = x^3 \in \mathbb{P}^3$

$f_1(x)|_{[\frac{1}{2},1]} = x^3 \in \mathbb{P}^3$, $f_1(+0) = f_1(-0) = 0$, $f_1'(+0) = f_1'(-0) = 0$

$f_1''(+0) = f_1''(-0) = 0 \Rightarrow f_1(x) \in C^2[-1,1] \Rightarrow f_1(x) \in S_{x,3}$ ✓

$f_2(x) := (x - \frac{1}{3})_+^3 = \begin{cases} (x-\frac{1}{3})^3 & x \geq \frac{1}{3} \\ 0 & x < \frac{1}{3} \end{cases}$

$f_2(x)|_{[-1,\frac{1}{2}]} \notin \mathbb{P}^3$, da „Sprung" $\Rightarrow f_2(x) \notin S_{x,3}$ ✓

$f_3(x) := -x + x^3 + 3x^5$

$f_3(x) \in \mathbb{P}^5 \wedge f_3(x) \notin \mathbb{P}^3$ (Grad zu hoch) $\Rightarrow f_3(x) \notin S_{x,3}$ ✓

$f_4(x) := \sum_{n=0}^{3} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$f_4(x)|_{[-1,-\frac{1}{2}]} = f_4(x)|_{[-\frac{1}{2},0]} = f_4(x)|_{[0,\frac{1}{2}]} = f_4(x)|_{[\frac{1}{2},1]} \in \mathbb{P}^3$

$f_4'(x) = a_1 + 2a_2 x + 3a_3 x^2$, $f_4''(x) = 2a_2 + 6a_3 x \Rightarrow f_4(x) \in C^2[-1,1]$

$\Rightarrow f_4(x) \in S_{x,3}$ ✓

$f_5(x) := x^3 e^x$

$f_5(x) \notin \mathbb{P}^3$, da e-Funktion keine Konst / kein Polynom
Auch als Potenzreihe ist exp(.) Pol. vom Grad ∞
$\Rightarrow \notin \mathbb{P}^3$. $\Rightarrow f_5(x) \notin S_{x,3}$ ✓

$f_6(x) := |x|^3 - |x + \frac{1}{3}|^2 = \begin{cases} x^3 - (x+\frac{1}{3})^2 & x \geq 0 \\ -x^3 - (x+\frac{1}{3})^2 & x < 0 \end{cases}$

$f_6(x)|_{[-1,-\frac{1}{2}]} = x^3 - x^2 - \frac{2}{3}x - \frac{1}{9} = f_6(x)|_{[-\frac{1}{2},0]} \in \mathbb{P}^3$

$f_6(x)|_{[0,\frac{1}{2}]} = -x^3 - x^2 - \frac{2}{3}x - \frac{1}{9} = f_6(x)|_{[\frac{1}{2},1]} \in \mathbb{P}^3$

$f_6(+0) = -\frac{1}{9} = f_6(-0)$, $f_6'(+0) = -\frac{2}{3} = f_6'(-0)$, $f_6''(+0) = -2 = f_6''(-0)$

$\Rightarrow f_6(x) \in C^2[-1,1] \Rightarrow f_6(x) \in S_{x,3}$ ✓

$f_7(x) := ||x|^3 - |x - \frac{1}{3}|^3| = \begin{cases} x^3 - (x-\frac{1}{3})^3 & (x \geq \frac{1}{3} \wedge x^3 - (x-\frac{1}{3})^3 \geq 0) \\ x^3 + (x-\frac{1}{3})^3 & (0 < x < \frac{1}{3} \wedge x^3 + (x-\frac{1}{3})^3 \geq 0) \leftarrow 0 < x < \frac{1}{6} \\ -x^3 + (x-\frac{1}{3})^3 & (x \leq 0 \wedge x^3 - (x-\frac{1}{3})^3 < 0) \leftarrow \text{immer falsch} \\ -x^3 - (x-\frac{1}{3})^3 & (0 < x < \frac{1}{3} \wedge x + (x-\frac{1}{3})^3 < 0) \leftarrow \frac{1}{6} < x < \frac{1}{3} \end{cases}$ ← Nullstelle $\frac{1}{6}$

$= \begin{cases} x^3 - (x-\frac{1}{3})^3 & x \geq \frac{1}{3} \vee x \leq 0 \\ x^3 + (x-\frac{1}{3})^3 & 0 < x \leq \frac{1}{6} \\ -x^3 - (x-\frac{1}{3})^3 & \frac{1}{6} < x < \frac{1}{3} \end{cases}$

$f_7(x)|_{[-1,-\frac{1}{2}]} \in \mathbb{P}^3$, $f_7(x)|_{[-\frac{1}{2},0]} \in \mathbb{P}^3$, $f_7(x)|_{[0,\frac{1}{2}]} \notin \mathbb{P}^3$, da kein Pol., da „Sprung" bei $\frac{1}{6}$ und $\frac{1}{3}$ ↓

$\Rightarrow f_7(x) \notin S_{x,3}$ ✓

$$f_8(x) := \begin{cases} (x+\frac{1}{2})^2 & -1 \le x < -\frac{1}{2} \\ 0 & -\frac{1}{2} \le x < \frac{1}{2} \\ -2(x-\frac{1}{2})^3 & \frac{1}{2} \le x < 1 \end{cases}$$

$f_8(x)|_{[-1,-\frac{1}{2}]} \in \mathbb{P}^2 \subset \mathbb{P}^3$, $f_8(x)|_{[-\frac{1}{2},0]} \in \mathbb{P}^0 \subset \mathbb{P}^3$, $f_8(x)|_{[0,\frac{1}{2}]} \in \mathbb{P}^3$

$(f_8(-\frac{1}{2}^+) = f_8(-\frac{1}{2}^-) = 0)$ $\quad$ $(f_8(\frac{1}{2}^+) = f_8(\frac{1}{2}^-) = 0)$

$f_8(x)|_{[\frac{1}{2},1]} \in \mathbb{P}^3$

$f_8'(-\frac{1}{2}^-) = 0 = f_8'(-\frac{1}{2}^+)$ $\quad$ $f_8'(\frac{1}{2}^+) = 0 = f_8'(+\frac{1}{2}^-)$

$f_8''(-\frac{1}{2}^-) = 2 \neq 0 = f_8''(-\frac{1}{2}^+)$ $\Rightarrow$ $f_8(x) \notin C^2[-1,1]$ $\Rightarrow$ $f_8(x) \notin S_{x,3}$ ✓

Numerik 1 Blatt 9 Julian Bergmann

Aufgabe 2

(10)

$$B_i^0(x) = \begin{cases} 1/x_{i+1} - x & , x \in [x_i, x_{i+1}] \\ 0 & , \text{sonst} \end{cases}$$

$$B_i^j(x) = \frac{x - x_i}{x_{i+j+1} - x_i} B_i^{j-1}(x) + \frac{x_{i+j+1} - x}{x_{i+j+1} - x_i} B_{i+1}^{j-1}(x)$$

n	0	1	2	3	4	5
x	0,5	1	2	3,1	4	4,2

$\xi = 2,6$

$B_i^0(x) = 0 \quad \forall\, i \in \{0,1,3,4\}$, $B_2^0 = \frac{1}{3,1-2} = \frac{1}{1,1}$ ✓

$B_0^1(x) = 0 + 0 = 0$ ✓

$B_1^1(x) = \frac{x_{i+j+1} - x}{x_{i+j+1} - x_i} B_2^0(x) = \frac{3,1 - x}{3,1 - 1} \cdot \frac{1}{1,1}$

$B_2^1(x) = \frac{x - x_i}{x_{i+j+1} - x_i} B_2^0(x) = \frac{x-2}{4-2} \cdot \frac{1}{1,1}$

$B_3^1(x) = 0$

$B_0^2(x) = \frac{x_{i+j+1} - x}{x_{i+j+1} - x_i} B_1^1(x) = \frac{3,1-x}{3,1-0,5} \frac{3,1-x}{3,1-1} \frac{1}{1,1}$

$B_1^2(x) = \frac{x - x_i}{x_{i+j+1} - x_i} B_1^1(x) + \frac{x_{i+j+1} - x}{x_{i+j+1} - x_i} B_2^1(x) = \frac{x-1}{4-1} \frac{3,1-x}{3,1-1} \frac{1}{1,1} + \frac{4-x}{4-1} \frac{x-2}{4-2} \frac{1}{1,1}$

$B_2^2(x) = \frac{x - x_i}{x_{i+j+1} - x_i} B_2^1(x) = \frac{x-2}{4,2-2} \frac{x-2}{4-2} \cdot \frac{1}{1,1}$

$B_0^3(x) = \frac{x - x_i}{x_{i+j+1} - x_i} B_0^2(x) + \frac{x_{i+j+1} - x}{x_{i+j+1} - x_i} B_1^2(x) = \frac{x-0,5}{4-0,5} \frac{3,1-x}{3,1-0,5} \frac{3,1-x}{3,1-1} \frac{1}{1,1}$

$+ \frac{4-x}{4-0,5} \left(\frac{x-1}{4-1} \frac{3,1-x}{3,1-1} \frac{1}{1,1} + \frac{4-x}{4-1} \frac{x-2}{4-2} \frac{1}{1,1} \right)$

$B_1^3(x) = \frac{x - x_i}{x_{i+j+1} - x_i} B_1^2(x) + \frac{x_{i+j+1} - x}{x_{i+j+1} - x_i} B_2^2(x) = \frac{x-1}{4,2-1} \left(\frac{x-1}{4-1} \frac{3,1-x}{3,1-1} \frac{1}{1,1} + \frac{4-x}{4-1} \frac{x-2}{4-2} \frac{1}{1,1} \right)$

$+ \frac{4,2-x}{4,2-1} \frac{x-2}{4,2-2} \frac{x-2}{4-2} \frac{1}{1,1}$

$B_0^4(x) = \frac{x - x_i}{x_{i+j+1} - x_i} B_0^3(x) + \frac{x_{i+j+1} - x}{x_{i+j+1} - x_i} B_1^3(x)$ ✓

$= \frac{x-0,5}{4,2-0,5} \left(\frac{x-0,5}{4-0,5} \frac{3,1-x}{3,1-0,5} \frac{3,1-x}{3,1-1} \frac{1}{1,1} + \frac{4-x}{4-0,5} \left(\left(\frac{x-1}{4-1} \frac{3,1-x}{3,1-1} \frac{1}{1,1} \right) + \frac{4-x}{4-1} \frac{x-2}{4,2} \frac{1}{1,1} \right) \right.$

$\left. + \frac{4,2-x}{4,2-0,5} \left(\frac{x-1}{4,2-1} \left(\frac{x-1}{4-1} \frac{3,1-x}{3,1-1} \frac{1}{1,1} + \frac{4-x}{4-1} \frac{x+2}{4-2} \frac{1}{1,1} \right) + \frac{4,2-x}{4,2-1} \frac{x-2}{4,2-2} \frac{x-2}{4-2} \frac{1}{1,1} \right) \right)$

$B_0^4(2,6) = 0,1378$ ✓