Numerik 1 Blatt 11 Julian Bergmann

Aufgabe 1

$F(x) = \int_0^1 \underbrace{\frac{1}{x+2}}_{f(x)} dx$, Gauß-Quadratur stimmt für pol. Funktionen mit Grad $2n+1$ exakt überein.

$\Rightarrow$ geforderd exakt Grad 5 $\Leftrightarrow n=2$

$$\int_a^b f(x)dx = \int_a^b \underbrace{w(x)}_{\text{Gewicht, hier } 1} \phi(x) dx = \int_a^b w(x) P_{n+1}(x) = \sum_{i=0}^{n} \phi(x_i) \alpha_i$$

$P_{n+1}(x) = P_3(x) = \frac{1}{2}(5x^3 - 3x) \Rightarrow x_{0,2} = \mp\sqrt{\frac{3}{5}}, x_1 = 0$ (Nullstellen v. P_3)

$\alpha_i = \int_{-1}^{1} \prod_{j=0, j\neq i}^{n} \frac{x-x_j}{x_i - x_j} dx$, $i = 0, \dots, n \Rightarrow \alpha_{0,2} = \frac{5}{9}$, $\alpha_1 = \frac{8}{9}$

Transformation: $\int_a^b f(x)dx = \frac{b-a}{2} \sum_{i=0}^{n} \alpha_i f(\frac{1}{2}x_i + \frac{1}{2})$

$$\Rightarrow \int_0^1 \frac{1}{x+2} dx = \frac{1}{2}\Big(\frac{5}{9} f(\frac{1}{2}\cdot(-\sqrt{\frac{3}{5}}) + \frac{1}{2}) + \frac{8}{9} f(\frac{1}{2}) + \frac{5}{9} f(\frac{1}{2}\sqrt{\frac{3}{5}} + \frac{1}{2})\Big)$$

$$= \frac{1}{2}\left(\frac{5}{9} f\left(\frac{5-\sqrt{15}}{10}\right) + \frac{8}{9} f(\frac{1}{2}) + \frac{5}{9} f\left(\frac{5+\sqrt{15}}{10}\right)\right)$$

$$= \frac{371}{915} \approx 0{,}4055$$

$$\int_0^1 \frac{1}{x+2} dx = \ln(1+2) - \ln(0+2) = 0{,}4055$$

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Aufgabe 2

a) Kriterium $\tilde{f}(x_0) \neq 0 \neq \tilde{f}'(x_0)$

$f(x) = e^x - 2x^3 \Rightarrow \tilde{f}(x) = e^x - 2x^3 - 6$

Nullstelle ist $f(x_n) = 6$

$\tilde{f}'(x) = e^x - 6x^2$

$x_0 = -1$

$\tilde{f}(-1) = -3{,}63 \neq 0$, $\tilde{f}'(-1) = -5{,}632 \neq 0$

$$x_{k+1} = x_k - \frac{\tilde{f}(x_k)}{\tilde{f}'(x_k)}$$

$$x_1 = -1 - \frac{e^{-1} - 2(-1)^3 - 6}{e^{-1} - 6(-1)^2} = -1{,}6449$$

$x_2 = -1{,}45201$, $x_3 = -1{,}42328$, $x_4 = -1{,}42287$

$x_5 = \underbrace{-1{,}422}_{\text{4 stellen bleiben gleich}}67 \Rightarrow f(x_5) = 6{,}000009 = 6$

b) ~~x_0~~ $x_0 = -1$, $x_1 = 4$, $x_{k+1} = x_k - \tilde{f}(x_k) \dfrac{x_k - x_{k+1}}{f(x_k) - f(x_{k+1})}$

$$x_2 = 4 - (e^4 - 2 \cdot 4^3 - 6) \frac{4+1}{e^4 - 2 \cdot 4^3 - e^{-1} + 2 \cdot (-1)^3} = -1{,}2397$$

$x_3 = -1{,}36815$, $x_4 = -1{,}43089$, $x_5 = -1{,}42234$

$x_6 = \underbrace{-1{,}422}_{\text{4 stellen bleiben gleich}}67 \Rightarrow f(x_6) = 5{,}99998 = 6$

c) $a_0 = -3$, $b_0 = 0$, Kriterium: $f(a_0) \cdot f(b_0) = -240 < 0$

$x_0 = \frac{1}{2}(a_0 + b_0) = -1{,}5$, $f(a_0) f(x_0) = 47 > 0 \Rightarrow a_1 = x_0$, $b_1 = b_0$

$x_1 = \frac{1}{2}(-1{,}5 + 0) = -0{,}75$, $f(a_1) f(x_1) = -5 < 0 \Rightarrow a_2 = a_1$, $b_2 = x_1$

$x_2 = \frac{1}{2}(-1{,}5 - 0{,}75) = -1{,}125$

$x_3 = -1{,}3125$, $x_4 = -1{,}40625$, $x_5 = -1{,}4531$

$x_6 = -1{,}4297$, $x_7 = -1{,}4179$, $x_8 = -1{,}42383$

$x_9 = -1{,}4209$, $x_{10} = -1{,}42236$, $x_{11} = -1{,}4231$

$x_{12} = -1{,}42272$, $x_{13} = \underbrace{-1{,}422}_{\text{4 stellen bleiben gleich}}55$

$f(x_{13}) = 5{,}9985 = 6$

$\Rightarrow$