

Aufgabe 9

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$$a) \mathcal{L}(x) = [\partial_m \phi^*(x)] [\partial^m \phi(x)] - m^2 \phi^*(x) \phi(x) + \lambda^2 [\phi^*(x) \phi(x)]^2$$

$$\frac{\partial \mathcal{L}(x)}{\partial \phi^*} = \cancel{\partial_m \partial^m \phi(x)} - m^2 \phi(x) + 2 \lambda^2 \phi^*(x) \phi(x) \quad \checkmark \quad E-L-GL?$$

$$\frac{\partial \mathcal{L}(x)}{\partial \phi} = \cancel{\partial} - m^2 \phi^*(x) + 2 \lambda^2 \phi^{*2} \phi(x) \quad \checkmark$$

$$b) \pi(x) = \frac{\partial \mathcal{L}(x)}{\partial(\partial^0 \phi)} = \frac{\partial (\partial_0 \phi^*(x) \partial^0 \phi(x) - \vec{\nabla} \phi^*(x) \vec{\nabla} \phi(x))}{\partial(\partial^0 \phi)} \quad \checkmark$$

$$= \cancel{\partial} \partial_0 \phi^*(x) + \cancel{\vec{\nabla} \phi^*(x)}$$

$$\pi^*(x) = \frac{\partial \mathcal{L}(x)}{\partial(\partial^0 \phi^*)} = \frac{\partial (\partial_0 \phi^*(x) \partial^0 \phi(x) + \cancel{\vec{\nabla} \phi^*(x) \vec{\nabla} \phi(x)})}{\partial(\partial^0 \phi^*)} \quad \checkmark$$

$$= \frac{\partial (\partial^0 \phi^*(x) \partial_0 \phi(x))}{\partial(\partial^0 \phi^*)} = \partial_0 \phi(x) \quad \checkmark$$

$$c) \mathcal{H} = \pi(\partial^0 \phi) + \pi^*(\partial^0 \phi^*) - \mathcal{L}(x)$$

$$= \partial_0 \phi^* \partial^0 \phi + \partial_0 \phi \partial^0 \phi^* - \partial_0 \phi^* \partial^0 \phi + \vec{\nabla} \phi^* \vec{\nabla} \phi + m^2 \phi^* \phi - \lambda^2 \phi^{*2} \phi^2$$

$$= \underbrace{\partial_0 \phi \partial^0 \phi^*}_{\partial_0 \phi \partial_0 \phi^* = \pi^* \pi} + \vec{\nabla} \phi^* \vec{\nabla} \phi + m^2 \phi^* \phi - \lambda^2 \phi^{*2} \phi^2 \quad \checkmark$$