Aufgabe 1 hQM WS 11/12

$$P = \int \frac{d^3q}{2w_q} q (a^+a + b^+b)$$

$$Q = \int \frac{d^3q}{2w_q} (a^+a - b^+b)$$

$$[a, a^+]_- = 2w_q \delta^3(q-q')$$

$$[b, b^+]_- = 2w_q \delta^3(q-q')$$

$$[\ldots]_- = 0$$

$$PQ = \int \frac{d^3q}{2w_q} q (a^+a + b^+b)(a^+a - b^+b)$$

$$a^+a(a^+a - b^+b) = a^+a a^+a - b^+b a^+a = (a^+a - b^+b)a^+a$$

$$b^+b(a^+a - b^+b) = a^+a b^+b - b^+b b^+b = (a^+a - b^+b)b^+b$$

$$\Rightarrow \quad PQ = \int \frac{d^3q}{2w_q} q (a^+a + b^+b)(a^+a - b^+b) = \int \frac{d^3q}{2w_q} (a^+a - b^+b)(a^+a + b^+b) q$$

$$= QP$$

Klausur höhere QM WS 11/12

Aufgabe 2

a) $I_a = \int_{-\infty}^{\infty} dx \underbrace{\frac{1}{x^4+1}}_{f}$

Polstellen Integrant:

$x_{1,2} = \pm\sqrt{i}$, $x_{3,4} = \pm i\sqrt{i}$

Wähle Weg über pos. $\mathbb{C}$-Bereich:

$a_1 = \sqrt{i}$, $a_2 = i\sqrt{i}$

$$I_a = 2\pi i \sum_a Res_a(f) = 2\pi i\left(\frac{1}{4(\sqrt{i})^3} + \frac{1}{4(i\sqrt{i})^3}\right)$$

$$= 2\pi i\left(\frac{1}{4i\sqrt{i}} + \frac{1}{4i^4\sqrt{i}}\right) = \frac{\pi}{2}\left(\frac{1}{\sqrt{i}} + \frac{i}{\sqrt{i}}\right) = \frac{\pi}{2}\left(-\sqrt{i}^3 + \sqrt{i}\right) = \frac{\pi}{\sqrt{2}}$$

b) $I_b = \int_{-\infty}^{\infty} d\omega \underbrace{\frac{e^{i\omega t}}{\omega^2 - a^2 + i\varepsilon}}_{f}$, Polstellen:

$\omega_{1,2} = \pm\sqrt{a^2 - i\varepsilon}$

Wähle Weg über pos $\mathbb{C}$-Bereich:

$a_1 = \sqrt{a^2 - i\varepsilon}$

$$I_b = 2\pi i\, Res_{a_1}(f) = 2\pi i \frac{e^{i\sqrt{a^2-i\varepsilon}\, t}}{2\sqrt{a^2 - i\varepsilon}} \xrightarrow[\varepsilon \to 0^+]{} \frac{e^{iat}}{a}\pi i$$

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Aufgabe 3

Lagrange-Dichte: $\mathcal{L}(\psi) = \bar\psi (i\gamma^\mu_\mu \partial_\mu - m)\psi$

$\psi' := e^{i\alpha_a \frac{\tau_a}{2}}\psi \quad , V: \psi \to \psi'$

$\psi' = \psi e^{i\alpha_a \frac{\tau_a}{2}}$

$\bar\psi' = \psi'^+ \gamma^0 = \psi^+ e^{-i\alpha_a \frac{\tau_a}{2}} \gamma^0 = \psi^+ \gamma^0 e^{-i\alpha_a \frac{\tau_a}{2}} = \bar\psi e^{-i\alpha_a \frac{\tau_a}{2}}$

$\Rightarrow \mathcal{L}(\psi') = \bar\psi e^{-i\alpha_a \frac{\tau_a}{2}} (i\gamma^\mu \partial_\mu - m)\psi e^{i\alpha_a \frac{\tau_a}{2}}$

$= \bar\psi (i\gamma^\mu \partial_\mu - m)\psi \underbrace{e^{i\alpha_a \frac{\tau_a}{2} - i\alpha_a \frac{\tau_a}{2}}}_{=1} \Rightarrow V$ invar.

$\psi' := e^{i\gamma_5 \alpha_a \frac{\tau_a}{2}}\psi \quad , \quad \alpha_a \frac{\tau_a}{2} := F \quad , A: \psi \to \psi'$

$= (1 + i\gamma_5 F - \frac{\gamma_5^2 F^2}{2} - \frac{i\gamma_5^3 F^3}{6} + \frac{\gamma_5^4 F^4}{24} + \dots)\psi$

$\quad\quad\quad\quad\quad\quad \mathbb{1} \quad\quad\quad \gamma_5 \quad\quad\quad \mathbb{1}$

$\bar\psi' = (\mathbb{1} - i\overset{\gamma_5^+ = \gamma_5}{\gamma_5} F - \frac{F^2}{2} + i\gamma_5 F^3 \cdot \frac{1}{6} + \frac{F^4}{24} + \dots)\psi^+ \gamma^0 = e^{-i\gamma_5 F}\psi^+ \gamma^0$

vertauschen mit γ^0 und $\gamma^\mu \partial_\mu$

$\Rightarrow$ betrachte γ_5

$\{\gamma^5, \gamma^\mu\} = 0$

$\Rightarrow \bar\psi' = \psi^+ \gamma^0 (\mathbb{1} + i\gamma_5 F - \frac{F^2}{2} - i\gamma_5 F^3 \cdot \frac{1}{6} + \frac{F^4}{24} + \dots)$

$= \psi^+ \gamma e^{i\gamma_5 F}$

$\bar\psi'(i\gamma^\mu \partial_\mu)\psi' = \bar\psi (i\gamma^\mu \partial_\mu) \underbrace{(\mathbb{1} - i\gamma_5 F - \frac{F^2}{2} + i\gamma_5 F^3 \frac{1}{6} + \dots)}_{e^{-i\gamma_5 F}} e^{i\gamma_5 F}\psi$

$= \bar\psi (i\gamma^\mu \partial_\mu)\psi$

Aber: $\bar\psi'(m)\psi' = \bar\psi(m) \underbrace{e^{i\gamma_5 F} e^{i\gamma_5 F}}_{\neq 1}\psi$

$\Rightarrow$ A nicht invariant!

Aufgabe 4

a) $\mathcal{L} = \overset{1}{\mathcal{L}_B} + \overset{2}{\mathcal{L}_{Mes}} + \overset{3}{\mathcal{L}_{int}}$

$= \overbrace{\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2}^{2} + \overbrace{\bar{\Psi}(i\gamma_\mu\partial^\mu - m)\Psi}^{1} + \overbrace{g_s\bar{\Psi}\phi\Psi}^{3}$

b) $\partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\Psi})} = 0 \quad , \quad \frac{\partial\mathcal{L}}{\partial(\bar{\Psi})} = i\gamma_\mu\partial^\mu\Psi + g_s\phi\Psi \overset{!}{=} 0$

$\partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\Psi)} - \frac{\partial\mathcal{L}}{\partial(\Psi)} = \partial_\mu(\bar{\Psi}\gamma_\mu i) + \bar{\Psi}m - g_s\bar{\Psi}\phi \overset{!}{=} 0$

$\partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} - \frac{\partial\mathcal{L}}{\partial(\phi)} = \frac{1}{2}\partial_\mu\partial^\mu\phi + m^2\phi - g_s\bar{\Psi}\Psi \overset{!}{=} 0$