2 Übungsblatt von Informatik 3 zum Mittwoch, den 4.5.2011

2.1 Aufgabe

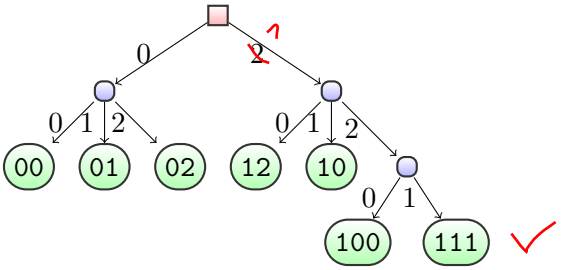
a) C_1: $\sum_i 3^{-n_i} = 5 \cdot 3^{-2} + 2 \cdot 3^{-3} = \frac{17}{27}$

C_2: $\sum_i 4^{-n_i} = 3 \cdot 4^{-2} + 3 \cdot 4^{-3} = \frac{7}{32}$

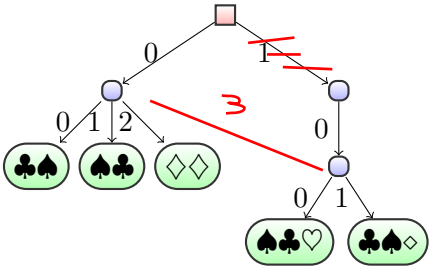
C_3: $\sum_i 2^{-n_i} = 3 \cdot 2^{-2} + 2 \cdot 2^{-3} = 1$

C_4: $\sum_i 3^{-n_i} = 1 \cdot 3^{-1} + 5 \cdot 3^{-3} + 2 \cdot 3^{-4} = \frac{44}{81}$

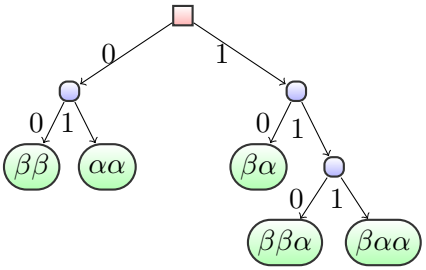
b) C_1:



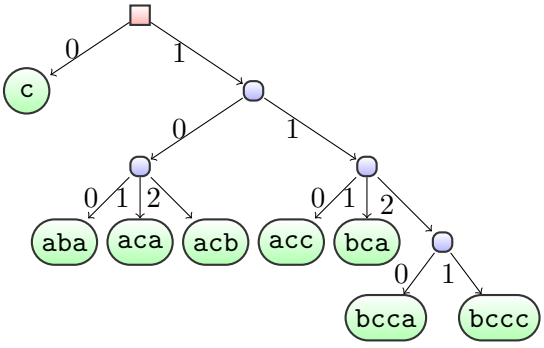
C_2:



C_3:



C_4:



2.2 Aufgabe

7,5/8

a) $q = 2$, $-\frac{\log(p_i)}{\log(q)} < n_i < -\frac{\log(p_i)}{\log(q)} + 1$:

	0	1	2	3	4	5
p(·)	0.25	0.03	0.14	0.15	0.17	0.26

$\Rightarrow$

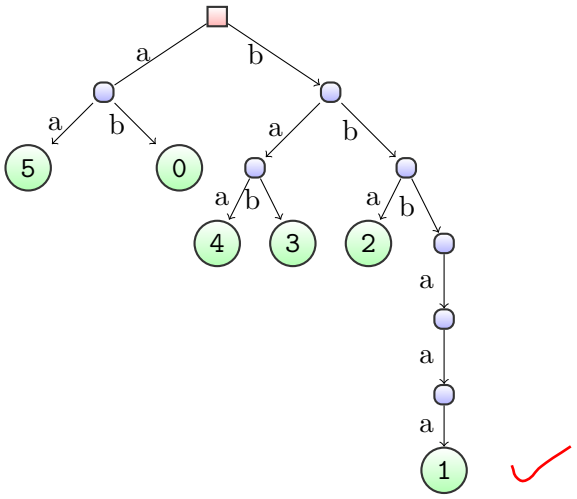
	5	0	4	3	2	1
p(·)	0.26	0.25	0.17	0.15	0.14	0.03
$-\frac{\log(p)}{\log(q)}$	1.943	2	2.556	2.737	2.837	5.059
ni	2	2	3	3	3	6

$\Rightarrow H(S) = -\sum_{i=1}^{6} p_i \log(p_i) = 2.3993$

$\Rightarrow \frac{H(S)}{\log(q)} + 1 = \frac{2.3993}{\log(2)} + 1 = 3.3993$

$\overline{\lambda}(\phi, S) = 0.26 \cdot 2 + 0.25 \cdot 2 + 0.17 \cdot 3 + 0.15 \cdot 3 + 0.14 \cdot 3 + 0.03 \cdot 6 = 2.58$

b)



Zunächst könnte man die 1 an die Stelle „bbb“ setzen. Dies ergebe schon $\overline{\lambda}(\phi, S) = 2.49$

(weiterhin decodierbar, da Präfixcode)

Noch näher käme man mit den Verfahren von Shannon oder Hoffmann.

c) a:

$E(\phi, S) = \frac{H(S)}{\log(q) \cdot \overline{\lambda}(\phi, S)} = \frac{2.3993}{\log(2) \cdot 2.58} = 0.93$

$\Rightarrow R(\phi, S) = 1 - E(\phi, S) = 1 - 0.848 = 0.152$

0.07

b:

$E(\phi, S) = \frac{H(S)}{\log(q) \cdot \overline{\lambda}(\phi, S)} = \frac{2.3993}{\log(2) \cdot 2.49} = 0.964$

$\Rightarrow R(\phi, S) = 1 - E(\phi, S) = 1 - 0.964 = 0.036$

2.3 Aufgabe

$$H(p_1, ..., p_{m-1}, q_1, q_2) = H(p_1, ..., p_m) + p_m \cdot H(\tfrac{q_1}{p_m}, \tfrac{q_2}{p_m})$$

$$\Rightarrow H(q_1, q_2) = H(p_m) + p_m \cdot H(\tfrac{q_1}{p_m}, \tfrac{q_2}{p_m})$$

$$\Rightarrow q_1 \log(q_1) + q_2 \log(q_2) = (q_1+q_2)\log(q_1+q_2) + (q_1+q_2) * \left(\tfrac{q_1}{q_1+q_2} \log\left(\tfrac{q_1}{q_1+q_2}\right) + \tfrac{q_2}{q_1+q_2} \log\left(\tfrac{q_2}{q_1+q_2}\right)\right)$$

$$\Rightarrow q_1 \log(q_1) + q_2 \log(q_2) = (q_1+q_2)\log(q_1+q_2) + q_1 \log(q_1) - q_1 \log(q_1+q_2) + q_2 \log(q_2) - q_2 \log(q_1+q_2)$$

$$\Rightarrow q_1 \log(q_1) + q_2 \log(q_2) = q_1 \log(q_1) + q_2 \log(q_2) \Rightarrow \checkmark$$