

Aufgabe 1

a)  $\partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \psi)} - \frac{\partial \mathcal{L}}{\partial \psi} = 0 - (i\gamma^\mu \partial_{\mu-m})\psi + g\gamma^\mu + V_\mu$

$\partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \bar{\psi})} - \frac{\partial \mathcal{L}}{\partial \bar{\psi}} = \bar{\psi} i\gamma^\mu - \bar{\psi} \cancel{m} - g\bar{\psi} \gamma^\mu V_\mu \stackrel{!}{=} 0$

b) Ergebnis vorhanden

Aufgabe 2

a)  $F(\vec{q}) = \int d^3r b \frac{e^{-ar}}{r} e^{i\vec{q}\vec{r}} = \int_0^{2\pi} d\varphi \int_0^\pi d\vartheta \int_0^\infty dr r^2 \sin(\vartheta) b \frac{e^{-ar}}{r} e^{iqr \cos(\vartheta)}$

$\frac{d \cos(\vartheta)}{d\vartheta} = -\sin(\vartheta) \Rightarrow F(\vec{q}) = \int_0^{2\pi} d\varphi \int_{-1}^1 d\cos(\vartheta) \int_0^\infty dr br \frac{\sin}{-\sin} e^{-ar} e^{iqr \cos \vartheta}$

$= 2\pi \int_{-1}^1 d\cos(\vartheta) \int_0^\infty dr br e^{-ar + iqr \cos(\vartheta)}$

$= 2\pi \int_0^\infty dr br e^{-ar} (e^{iqr} - e^{-iqr}) = 2\pi \int_0^\infty dr br e^{-r} (e^{iqr} - e^{-iqr})$

$\int r e^{ar} dr = \frac{ar-1}{a^2} e^{ar} \Rightarrow F(\vec{q}) = 2\pi b \left[ \frac{-a+iq}{(-a+iq)^2} e^{-a-iq} - \frac{-a-iq}{(-a-iq)^2} e^{-a+iq} \right]$

$\Rightarrow F(\vec{q}) = 2\pi \left[ \frac{(-a+iq)r-1}{(-a+iq)^2} e^{-a-iq} - \frac{(-a-iq)r-1}{(-a-iq)^2} e^{-a+iq} \right]$   
 $= 2\pi \left( 0 - 0 - \frac{-1}{(-a+iq)^2} + \frac{-1}{(-a-iq)^2} \right)$   
 $= 2\pi \left( \frac{1}{(iq-a)^2} - \frac{1}{(a+iq)^2} \right)$

b)  $(\partial_\mu \partial^\mu - m^2) \psi = 0$  (KG),  $\partial_\mu \psi = 0 \Leftrightarrow (-\nabla^2 - m^2) \psi \stackrel{!}{=} 0$

$\nabla^2|_r = \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r}, -\nabla^2 \psi = a^2 b^2 e^{-ar} r^3$   
 $-m^2 \psi = m^2 b^2 e^{-ar}$

... auch irgendwie falsch...

# Aufgabe 3

- a) siehe Kl. WS 11/12 Aufg. 4
- b) "
- c) Existiert auch ...

# Aufgabe 4

a) Pole nur durch  $\frac{1}{k^2 - m^2 + i\epsilon} = \frac{1}{k_0^2 - k_1^2 - k_2^2 - k_3^2 - m^2 + i\epsilon}$

b)  $\Rightarrow k_{0,2} = \pm \sqrt{k_1^2 + k_2^2 + k_3^2 + m^2 - i\epsilon}$

The diagram shows the complex plane with a horizontal real axis (Re) and a vertical imaginary axis (Im). Two poles are marked on the imaginary axis at  $\pm i\sqrt{F - i\epsilon}$ . A rectangular contour is drawn in the upper half-plane, enclosing the pole at  $i\sqrt{F - i\epsilon}$ . The contour is labeled with a circled '1'.

Weg über pos  $\mathbb{R}$  Bereich  
 Falluntersuch wegen unprob. Zähler nicht notwendig!

$\Theta(1 - |k|) \Rightarrow$  aber da  $|k|$  abh. v.  $k_1 - k_3$  und in 1. Int nur  $k_0$ , hier vernachl.

The diagram shows a circle in the complex plane centered at the origin. A shaded sector is drawn in the first quadrant, bounded by the positive real axis and a line at an angle. The axes are labeled Re and Im.

c)  $\int dk_0 \frac{1}{k^2 - m^2 + i\epsilon} = 2\pi i \left( \frac{1}{2\sqrt{k_1^2 + k_2^2 + k_3^2 + m^2 - i\epsilon}} \right)$

$\Rightarrow m_0 = m_0 + 48iG \int \frac{d^3k}{(2\pi)^3} \Theta(1 - |k|) \frac{2\pi i}{2\pi} \frac{1}{2\sqrt{k_1^2 + k_2^2 + k_3^2 + m^2 - i\epsilon}}$

$= m_0 - 48G \int_0^{2\pi} d\varphi \int_0^\pi d\alpha \int_0^1 dr r^2 \sin(\alpha) \frac{1}{(2\pi)^3} \frac{1}{2\sqrt{r^2 + m^2 - i\epsilon}}$

$= m_0 - 48G \frac{2\pi}{(2\pi)^3} \int_{-1}^1 dr \frac{r^2}{\sqrt{r^2 + m^2 - i\epsilon}} \quad (\text{Res. Satz: } \int_{-\infty}^{\infty} \frac{1}{\sqrt{m^2 + i\epsilon}})$

$= m_0 - 48G \frac{2\pi}{(2\pi)^3} \cdot 2\pi i \left( \frac{-m^2 + i\epsilon}{m^2 + i\epsilon} \frac{1}{m^2 - i\epsilon + m^2 + i\epsilon} \right) = 0$

auch seltsam

Aufgabe 5

Nach VL:  $C^t C = C C^t = 1$

$$C^t C C^{-1} = 1 C^{-1} \Leftrightarrow C^t = C^{-1}$$

EXP:  $C = i \gamma^2 \gamma^0 = \begin{pmatrix} 0 & -i\sigma^2 \\ -i\sigma^2 & 0 \end{pmatrix}$ ,  $\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$$C^* = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \text{Achtung!}$$

$C^t = C^{-1}$  da reell

$$\begin{pmatrix} 0 & 0 & 0 & 1 & | & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & | & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & | & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & | & 0 & 0 & 0 & 1 \end{pmatrix} \Leftrightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & | & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & | & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & | & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & | & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$C^{-1} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} = -C$$

$$T = - \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix} = \begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{pmatrix}, \quad \text{Achtung!}$$

~~Wichtig!~~ da T reell komplex

$$T^{-1} \Leftrightarrow \begin{pmatrix} 0 & i & 0 & 0 & | & 1 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 & | & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & i & | & 0 & 0 & 1 & 0 \\ 0 & 0 & -i & 0 & | & 0 & 0 & 0 & 1 \end{pmatrix} \Leftrightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & | & 0 & i & 0 & 0 \\ 0 & 1 & 0 & 0 & | & -i & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & | & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 1 & | & 0 & 0 & -i & 0 \end{pmatrix}$$

$$\Rightarrow T^{-1} = \begin{pmatrix} -\sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix} = T$$

$$T^t = \begin{pmatrix} 0 & -i & 0 & 0 \\ +i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & +i & 0 \end{pmatrix} = \begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{pmatrix} = T$$