Klausur 2 HQM WS 9/10

Aufgabe 1

a) $\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu\bar\Psi)} - \frac{\partial\mathcal{L}}{\partial\bar\Psi} = 0 - (i\gamma^\mu\partial_\mu - m)\Psi + g\gamma^\mu\Psi V_\mu \stackrel{!}{=} 0$

$\partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\Psi)} - \frac{\partial\mathcal{L}}{\partial\Psi} = \bar\Psi i\gamma^\mu - \bar\Psi m - g\bar\Psi\gamma^\mu V_\mu \stackrel{!}{=} 0$

b) Ergebnis vorhanden

Aufgabe 2

a) $F(\vec q) = \int d^3r\, b\frac{e^{-ar}}{r} e^{i\vec q\vec r} = \int_0^{2\pi} d\varphi \int_0^\pi d\vartheta \int_0^\infty dr\, r^2\sin(\vartheta)\, b \frac{e^{-ar}}{r} e^{iqr\cos(\vartheta)}$

$\frac{d\cos(\vartheta)}{d\vartheta} = -\sin(\vartheta) \quad\Rightarrow\quad F(\vec q) = \int_0^{2\pi} d\varphi \int_1^{-1} d\cos(\vartheta)\int_0^\infty dr\, br \frac{\sin}{-\sin} e^{-ar} e^{iqr\cos\vartheta}$

$= 2\pi \int_{-1}^{1} d\cos(\vartheta)\int_0^\infty dr\, b e^{-ar+iqr\cos(\vartheta)}\, r$

$= 2\pi\int_0^\infty dr\, br\, e^{-ar}(e^{iqr} - e^{-iqr}) = 2\pi\int_0^\infty dr\, br\, e^{r(-a+iq)} - br\, e^{-r(a+iq)}$

$\int re^{ar}dr = \frac{ar-1}{a^2}e^{ar}$

$\Rightarrow F(\vec q) = 2\pi\left[\frac{(-a+iq)r-1}{(-a+iq)^2}\underbrace{e^{(-a+iq)r}}_{\substack{r\to\infty \\ a-iq>0 \\ 0}} - \frac{(-a-iq)r-1}{(-a-iq)^2}\underbrace{e^{(-a-iq)r}}_{\substack{r\to\infty \\ a+iq>0 \\ 0}}\right]_0^\infty$

$= 2\pi\left(0 - 0 - \frac{-1}{(-a+iq)^2} + \frac{-1}{(-a-iq)^2}\right)$

$= 2\pi\left(\frac{1}{(iq-a)^2} - \frac{1}{(a+iq)^2}\right)$

b) $(\partial_\mu\partial^\mu - m^2)\rho = 0$ (KG). $\partial_0\rho = 0 \Leftrightarrow (-\vec\nabla^2 - m^2)\rho \stackrel{!}{=} 0$

$\vec\nabla^2\big|_r = \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}$, $-\vec\nabla^2\rho = a^2b^2e^{-ar}r^3$

$-m^2\rho = m^2b^2\frac{e^{-ar}}{r}$

... auch irgendwie falsch...

Aufgabe 3

a) siehe Kl. WS11/12 Aufg. 4

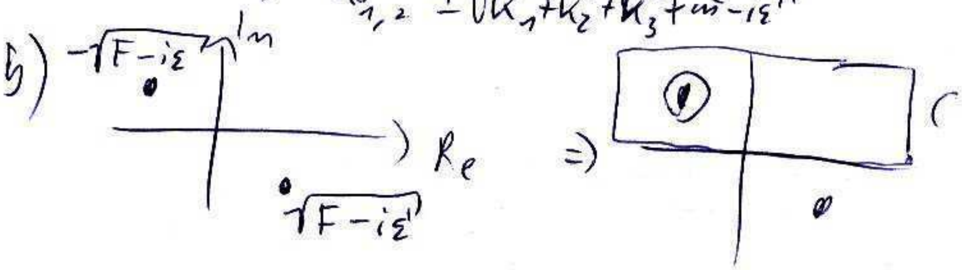
b) ''

c) Existiert auch ...

Aufgabe 4

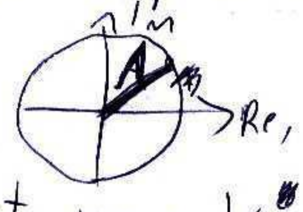
a) Pole nur durch $\frac{1}{k^2-m^2+i\varepsilon'} = \frac{1}{k_0^2-k_1^2-k_2^2-k_3^2-m^2+i\varepsilon'}$

$\Rightarrow k_{0_{1,2}} = \pm\sqrt{k_1^2+k_2^2+k_3^2+m^2-i\varepsilon'}$

b) 

Weg über pos Im Bereich

Fallunterscheidung wegen unprob. Zähler nicht notwendig,

$\Theta(1-|k|) \Rightarrow$ 

aber da $|k|$ abh. v. $k_1 - k_3$

und in 1. Int nur k_0, hier vernachl.

c) $\int dk_0 \frac{1}{k^2-m^2+i\varepsilon'} = 2\pi i\left(\frac{1}{2\sqrt{k_1^2+k_2^2+k_3^2+m^2-i\varepsilon'}}\right)$

$\Rightarrow m = m_0 + 48iG\int\frac{d^3k}{(2\pi)^3}\Theta(1-|k|)\frac{2\pi i}{2\pi}\frac{1}{2\sqrt{k_1^2+k_2^2+k_3^2+m_2^2-i\varepsilon'}}$

$= m_0 - 48G\int_0^{2\pi}d\varphi\int_0^{\pi}d\vartheta\int_{-1}^{1}dr\, r^2\sin(\vartheta)\frac{1}{(2\pi)^3}\frac{1}{2\sqrt{r^2+m^2-i\varepsilon'}}$

$= m_0 - 48G\frac{2\pi}{(2\pi)^3\cdot 2}\int_{-1}^{1}dr\frac{r^2}{\sqrt{r^2+m^2-i\varepsilon'}}$ (Res.-Satz: NS $\sqrt{m^2+i\varepsilon}$)

$= m_0 - 48G\frac{2\pi}{(2\pi)^3\cdot 2}\cdot 2\pi i\left(\frac{-m^2+i\varepsilon'}{\underbrace{\sqrt{m^2+i\varepsilon'}\sqrt{-m^2-i\varepsilon'+m^2+i\varepsilon'}}_{=0}}\right)$

auch seltsam

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Aufgabe 5

Nach VL: $C^+ C = C C^+ = 1$

$C^+ C C^{-1} = 1 C^{-1} \Leftrightarrow C^+ = C^{-1}$

Exp: $C = i\gamma^2\gamma^0 = \begin{pmatrix} 0 & -i\sigma^2 \\ -i\sigma^2 & 0 \end{pmatrix}$, $\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$C^* = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$ ~~[illegible]~~

$C^+ = C^{-1}$; da reell
$\left(\begin{array}{cccc|cccc} 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right) \Leftrightarrow \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{array}\right)$

$C^{-1} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} = -C$

$T = -\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix} = \begin{pmatrix} 0 & i & & \\ -i & 0 & & \\ & & 0 & i \\ & & -i & 0 \end{pmatrix}$, ~~$T^+ = \ldots = -T$~~

~~$T^+ = -T$ da T rein komplex~~

$T^{-1} \Leftrightarrow \left(\begin{array}{cccc|cccc} 0 & i & 0 & 0 & 1 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & i & 0 & 0 & 1 & 0 \\ 0 & 0 & -i & 0 & 0 & 0 & 0 & 1 \end{array}\right) \Leftrightarrow \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & i & 0 & 0 \\ 0 & 1 & 0 & 0 & -i & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & +i \\ 0 & 0 & 0 & 1 & 0 & 0 & -i & 0 \end{array}\right)$

$\Rightarrow T^{-1} = \begin{pmatrix} -\sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix} = T$

$T^+ = \begin{pmatrix} 0 & -i & & 0 \\ +i & 0 & & \\ 0 & & 0 & -i \\ & & +i & 0 \end{pmatrix}^T = \begin{pmatrix} 0 & i & & 0 \\ -i & 0 & & \\ 0 & & 0 & i \\ & & -i & 0 \end{pmatrix} = T$