

9.7

a) $a_1 = (1, 1, 1, 0, 0, 1, 0, 1, 0, 0, 0, 1, 1) \sim 1 + x + x^2 + x^5 + x^7 + x^{10} + x^{13} + x^{14}$
 $(x^{14} + x^{13} + x^{11} + x^7 + x^5 + x^2 + x + 1) : (x^4 + x + 1) = x^{10} + x^9 + x^5 + x^3 + x^2 + 1$
 $-x^{14} - x^{13} + x^{10}$
 $x^3 + x^{10} + x^7 + x^5 + x^2 + x + 1$
 $-x^{13} - x^{10} - x^9$
 $x^7 + x^7 + x^5 + x^2 + x + 1$
 $-x^9 + x^6 - x^5$
 $x^7 + x^6 + x^2 + x + 1$
 $-x^7 - x^4 - x^3$
 $x^6 + x^4 + x^3 + x^2 + x + 1$
 $-x^6 + x^3 - x^2$
 $x^4 + x + 1$

$\sim (1, 0, 1, 1, 0, 1, 0, 0, 0, 1, 1)$

b) $a_2 = (1, 1, 0, 1, 1, 1, 1, 1, 0, 1, 0, 1, 0) \sim 1 + x + x^3 + x^4 + x^5 + x^6 + x^7 + x^9 + x^{11} + x^{13}$
 $(x^{13} + x^{11} + x^9 + x^7 + x^6 + x^5 + x^4 + x^3 + x + 1) : (x^4 + x + 1) = x^9 + x^7 + x^6 + x^4 + x^3 + 1$
 $-x^{13} - x^{10} - x^9$
 $x^{11} + x^{10} + x^7 + x^6 + x^5 + x^4 + x^3 + x + 1$
 $-x^{11} - x^8 - x^7$
 $x^{10} + x^8 + x^6 + x^5 + x^4 + x^3 + x + 1$
 $-x^{10} - x^7 - x^6$
 $x^8 + x^7 + x^5 + x^4 + x^3 + x + 1$
 $-x^8 + x^5 + x^4$
 $x^7 + x^3 + x + 1$
 $-x^7 + x^4 - x^3$
 $x^4 + x + 1$

$\sim (1, 0, 0, 1, 1, 0, 1, 1, 0, 1, 0)$

c) $a_3 = (2, 0, 2, 2, 1, 2, 0, 1, 0, 2, 1, 0, 0) \sim 2 + 2x^2 + 2x^3 + x^4 + 2x^5 + x^7 + 2x^9 + 2x^{10} + x^{11}$
 $(x^{11} + 2x^{10} + 2x^9 + x^7 + 2x^5 + x^4 + 2x^3 + 2x^2 + 2) : (x^3 + 2x^2 + 1) = x^8 + 2x^6 + x^5 + 2x^4 + x^2 + 2$
 $-x^{11} - 2x^{10} - x^8$
 $2x^9 + 2x^8 + x^7 + 2x^5 + x^4 + 2x^3 + 2x^2 + 2$
 $-2x^9 - 2x^6 - 2x^6$
 $x^8 + x^7 + 2x^6 + 2x^5 + x^4 + 2x^3 + 2x^2 + 2$
 $-x^8 - 2x^7 - x^5$
 $2x^7 + 2x^6 + x^5 + x^4 + 2x^3 + 2x^2 + 2$
 $-2x^7 - x^6 - 2x^4$
 ~~$x^5 + 2x^4 + 2x^3 + 2x^2 + 2$~~
 $-x^5 - 2x^4 - x^2$
 $2x^3 + 2x^2 + 2$

$\sim (2, 0, 1, 0, 2, 1, 2, 0, 1, 0, 0, 0)$

d) $a_4 = (2, 1, 1, 2, 1, 0, 0, 0, 2, 1, 0, 1, 1, 1) \sim 2 + x + x^2 + 2x^3 + x^4 + 2x^6 + x^8 + x^{11} + x^{12} + x^{13} + x^{14}$
 $(x^{14} + x^{13} + x^{12} + x^{11} + x^9 + 2x^8 + x^4 + 2x^3 + x^2 + x + 2) : (x^3 + 2x^2 + 1) = x^{11} + 2x^{10} + x^7 + 2x^6 + x^5 + x^3 + x + 2$
 $-x^{14} - 2x^{13} - x^{11}$
 $2x^{13} + x^{12} + x^9 + 2x^8 + x^4 + 2x^3 + x^2 + x + 2$
 $-2x^{13} - x^{12} - 2x^{10}$
 $x^{10} + x^9 + 2x^8 + x^4 + 2x^3 + x^2 + x + 2$
 $-x^{10} - 2x^9 - x^7$
 $2x^9 + 2x^8 + 2x^7 + x^4 + 2x^3 + x^2 + x + 2$
 $-2x^9 - 2x^8 - 2x^6$
 $x^8 + 2x^7 + x^6 + x^4 + 2x^3 + x^2 + x + 2$
 $-x^8 - 2x^7 - x^5$
 ~~$x^6 + 2x^5 + x^4 + 2x^3 + x^2 + x + 2$~~
 $-x^6 - 2x^5 - x^3$

$\sim (2, 1, 0, 1, 0, 1, 2, 1, 0, 0, 2, 1)$

9.2

$$a) h(x) = \frac{x^6 - 1}{g(x)} = (x^6 - 1) : (x^3 + 2x^2 + 2x + 1) = x^3 + 3x^2 + 2x + 4$$

$$\begin{array}{r} x^6 - 1 \\ -x^6 - 2x^5 - 2x^4 - x^3 \\ \hline 3x^5 + 3x^4 + 4x^3 - 1 \\ -3x^5 + x^4 - x^3 - 3x^2 \\ \hline 2x^4 + 3x^3 + 2x^2 - 1 \\ -2x^4 - 4x^3 - 4x^2 - 2x \\ \hline 4x^3 + 3x^2 + 3x - 1 \\ -4x^3 + 3x^2 + 3x - 1 \\ \hline 0 \end{array}$$

$$b) G = \begin{pmatrix} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 2 & 1 \end{pmatrix}, \quad H = \begin{pmatrix} 0 & 0 & 1 & 3 & 2 & 4 \\ 0 & 1 & 3 & 2 & 4 & 0 \\ 1 & 3 & 2 & 4 & 0 & 0 \end{pmatrix}$$

$$c) 1) (1, 4, 2) \cdot \begin{pmatrix} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 2 & 1 \end{pmatrix} = (1, 1, 2, 3, 3, 2)$$

$$2) (1 + 4x + 2x^2)(1 + 2x + 2x^2 + x^3) = (1 + 2x^2 + 2x^2 + x^3 + 4x + 8x^2 + 8x^3 + 4x^4 + 2x^2 + 4x^3 + 4x^4 + 2x^5)_{\mathbb{F}_5}$$

$$= (1 + 6x + 12x^2 + 13x^3 + 8x^4 + 2x^5)_{\mathbb{F}_5}$$

$$= 1 + x + 2x^2 + 3x^3 + 3x^4 + 2x^5 \sim (1, 1, 2, 3, 3, 2)$$

\Rightarrow gleich!

$$d) 1) \begin{pmatrix} 0 & 0 & 1 & 3 & 2 & 4 \\ 0 & 1 & 3 & 2 & 4 & 0 \\ 1 & 3 & 2 & 4 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 4 \\ 3 \\ 2 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 25 \\ 25 \\ 30 \end{pmatrix}_{\mathbb{F}_5} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 4 \\ 4 \\ 3 \\ 2 \\ 2 \\ 3 \end{pmatrix} \text{ ist Kanalcodewort}$$

$$(3x^5 + 2x^4 + 2x^3 + 3x^2 + 4x + 4) : (x^3 + 2x^2 + 2x + 1) = 3x^2 + x + 4 \Rightarrow \underline{(4, 1, 3)}$$

$$\begin{array}{r} 3x^5 + 2x^4 + 2x^3 + 3x^2 + 4x + 4 \\ -3x^5 - x^4 - x^3 - 3x^2 \\ \hline x^4 + x^3 + 4x + 4 \\ -x^4 + 2x^3 - 2x^2 + x \\ \hline 3x^3 + 3x^2 + 3x + 4 \\ -4x^3 + 3x^2 - 3x - 4 \\ \hline 0 \end{array}$$

$$2) (x^3 + 3x^2 + 2x + 4) \cdot (3x^5 + 2x^4 + 2x^3 + 3x^2 + 4x + 4) =$$

$$= (3x^8 + 2x^7 + 2x^6 + 3x^5 + 4x^4 + 4x^3 + 9x^2 + 6x^6 + 6x^5 + 9x^4 + 12x^3 + 12x^2 + 6x^6 + 4x^5 + 4x^4 + 6x^3 + 8x^2 + 8x + 12x^5 + 8x^4 + 8x^3 + 12x^2 + 16x + 16)_{\mathbb{F}_5}$$

$$= (1 + 4x + 2x^2 + 4x^6 + x^7 + 3x^8)_{\mathbb{F}_5} = \underbrace{(4 + x + 3x^2)}_{(4, 1, 3)} (x^6 - 1)_{\mathbb{F}_5} = 0_{\mathbb{F}_5} \Rightarrow \text{Kanalcodewort.}$$

$$1) \begin{pmatrix} 0 & 0 & 1 & 3 & 2 & 4 \\ 0 & 1 & 3 & 2 & 4 & 0 \\ 1 & 3 & 2 & 4 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \\ 3 \\ 4 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 27 \\ 36 \\ 34 \end{pmatrix}_{\mathbb{F}_5} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} = 2H_3$$

$$\Rightarrow \text{gestört, neues Wort: } (3, 3, 3, 4, 4, 1) - (0, 0, 2, 0, 0, 0) = (3, 3, 1, 4, 4, 1)$$

$$(x^5 + 4x^4 + 4x^3 + x^2 + 3x + 3) : (x^3 + 2x^2 + 2x + 1) = x^2 + 2x + 3 \Rightarrow \underline{(3, 2, 1)}$$

$$\begin{array}{r} x^5 + 4x^4 + 4x^3 + x^2 + 3x + 3 \\ -x^5 - 2x^4 - 2x^3 - x^2 \\ \hline 2x^4 + 2x^3 + 3x + 3 \\ -2x^4 + 4x^3 - 4x^2 - 2x \\ \hline 3x^3 + x^2 + 8x + 3 \\ -3x^3 - x^2 - x - 3 \\ \hline 0 \end{array}$$

$$2) (x^3 + 3x^2 + 2x + 4) \cdot (x^5 + 4x^4 + 4x^3 + x^2 + 3x + 3) = (x^8 + 4x^7 + 4x^6 + x^5 + 3x^4 + 3x^3 + 3x^7 + 12x^6 + 12x^5 + 3x^4 + 9x^3 + 9x^2 + 2x^6 + 8x^5 + 8x^4 + 2x^3 + 6x^2 + 6x + 4x^5 + 16x^4 + 16x^3 + 4x^2 + 12x + 12)_{\mathbb{F}_5}$$

$$= (2 + 3x + 4x^2 + 3x^6 + 2x^7 + x^8)_{\mathbb{F}_5}$$

$$= \underbrace{(3 + 2x + x^2)}_{(3, 2, 1)} (x^6 - 1) + \underbrace{4x^3 + x^4 + 2x^5 + 3x^2}_{\text{Syndrom}} \leftarrow \text{Stelle d. Störung}$$