$333441 \sim \hat{v}(x)$

$$\frac{\hat{v}(x)h(x)}{x^6-1} = \frac{v(x)h(x) + e(x)h(x)}{x^6-1} = p(x) \; R \; q(x)$$

BCH-Code: $n=15,\ k=7,\ d=5 \quad g(x) = x^8 + x^7 + x^6 + x^4 + 1, \ \alpha$ NS

$M(x) = x^4 + x + 1$ Erw. Kp. $GF(2^4)$ $\quad \vartheta = 6 \Rightarrow \tau = 2$

$\hat{v} = (101001111011111)$

$\tilde{v}(x) = 1 + x^2 + x^3 + x^6 + x^7 + x^8 + x^{11} + x^{11} + x^{12} + x^{13} + x^{14}$

1. $\hat{v} = 0 \bmod g \Rightarrow$ nein

2. Syndrom berechnen: $\begin{pmatrix} S_1 & S_2 \\ S_2 & S_3 \end{pmatrix} \begin{pmatrix} \sigma_2 \\ \sigma_1 \end{pmatrix} = \begin{pmatrix} S_3 \\ S_4 \end{pmatrix}$

(Anzahl NS aneinanderhängend $+2 = \vartheta$
Hier: $\alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^6$, $\quad \vartheta = d+1$)

$S_1 = \hat{v}(\alpha) = 1 + \alpha^2 + \alpha^5 + \dots + \alpha^{19} = \alpha^4$

$S_{2j} = S_j^2 \Rightarrow S_2 = S_1^2 = \alpha^8, \quad S_4 = S_2^2 = \alpha^{16} = \alpha$

$S_3 = \hat{v}(\alpha^3) = 1 + \alpha^6 + \alpha^{15} + \dots = \alpha$

$$\Rightarrow \begin{pmatrix} \alpha^4 & \alpha^8 \\ \alpha^8 & \alpha \end{pmatrix} \begin{pmatrix} \sigma_2 \\ \sigma_1 \end{pmatrix} = \begin{pmatrix} \alpha \\ \alpha \end{pmatrix}$$

$\Rightarrow \alpha^4\sigma_2 + \alpha^8\sigma_1 = \alpha \quad \Rightarrow \alpha^8\sigma_1 = \alpha^4\sigma_2 + \alpha \Rightarrow \sigma_1 = \alpha^{11}\sigma_2 + \alpha^8$

$\alpha^8\sigma_2 + \alpha\sigma_1 = \alpha \quad \Rightarrow \alpha^8\sigma_2 + \alpha^{12}\sigma_2 + \alpha^9 = \alpha$

$\Rightarrow \sigma_1 = \alpha^4, \quad \sigma_2 = \alpha^9 \quad \Rightarrow \sigma(x) = x^2 + \sigma_1 x + \sigma_2$

$= x^2 + \alpha^4 x + \alpha^9$

$\stackrel{!}{=} (x + x_1)(x + x_2) \Rightarrow x_1 = \alpha^{11}, \ x_2 = \alpha^{13}$

$\Rightarrow e(x) = x^{11} + x^{13}$

$\Rightarrow v(x) = \hat{v}(x) + e(x) \Rightarrow v = (101001111010101)$

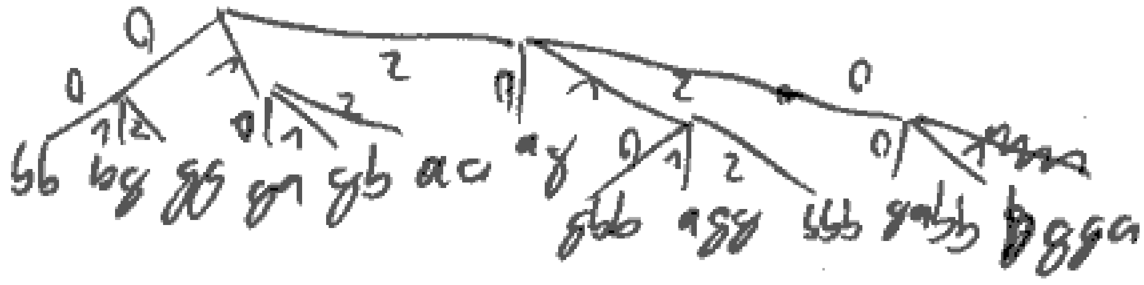
$C_3 = \{bb, bg, gbb, gg, ga, gb, agg\}$

$\sum_{i=1}^{m} q^{-n_i} = \frac{17}{27} \le 1$

$\Rightarrow \{00, 01, 02, 10, 11, 120, 121\}$

$C_3' = C_3 \cup \{aa, ag, bbb, gabb, bgga\}$

$\frac{17}{27} + \frac{2}{9} + \frac{1}{27} + \frac{2}{81} = \frac{74}{81}$



$\Rightarrow \{00, 01, 02, 10, 11, 12, 20, 210, 211, 212, 2200, 2201\}$

$I(x) = \log\left(\frac{1}{p(x)}\right)$, $H(S) = \sum_{a \in A} p(a) I(a) = -\sum_{a \in A} p(a) \log(p(a))$

Aufg:

a)

a	b	c	d	e	f
$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{1}{20}$

$\Rightarrow H(S) = 2{,}06$

b)

a	b	c	d	e	f
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$\Rightarrow H(S) = \max$ $(2{,}58)$

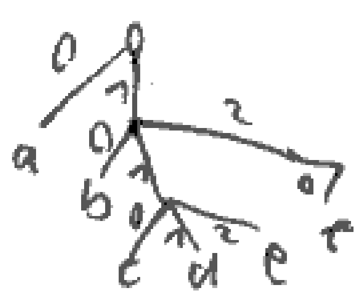
Shannon: 1) $\forall$ eind. Code $\varphi: A \to B^*$ $|B| = q$: $\bar{\lambda}(\varphi, S) \ge \frac{H(S)}{\log(q)}$

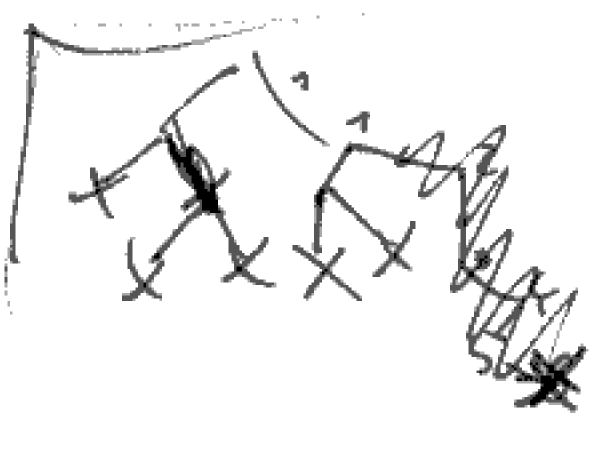
2) $\exists$ eind. dec. Code $\psi: A \to B^*$ mit $\bar{\lambda}(\psi, S) \le \frac{H(S)}{\log(q)} + 1$

$\Rightarrow -\frac{\log(p_i)}{\log(q)} \le n_i \le -\frac{\log(p_i)}{\log(q)} + 1$ wählen

Aufg: $\Rightarrow |B| = 3$

	a	b	c	d	e	f
p	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{1}{20}$
$-\frac{\log(p_i)}{\log(q)}$	0,63	1,465	2,096	2,096	2,72	2,727
n:	1	2	3	3	3	3





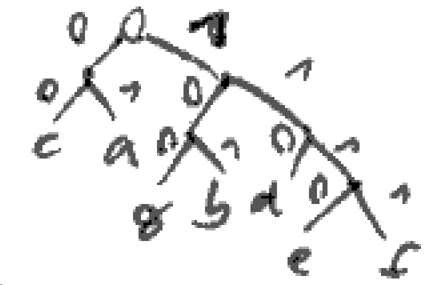
1)

a	b	c	d	e	f	g
0,18	0,1	0,4	0,08	0,05	0,05	0,14

→

c	a	g	b	d	e	f
0,4	0,18	0,14	0,1	0,08	0,05	0,05
0	0	1	1	1	1	1
0	1	0	0	1	1	1
		0	1	0	1	1
					0	1

Shannon, $q=2$



$$H(S) = -\sum_{a\in A} p(a)\log_2(p(a)) = 2{,}427$$

$$\bar\lambda = \sum_a \lambda(a)\cdot p(a) = 2{,}52$$

$$\Rightarrow E = \frac{H(S)}{\bar\lambda \cdot \log_2(q)} = \frac{2{,}427}{2{,}52} = 0{,}963$$

$R = 0{,}037$

2) Huffmann $q=2$

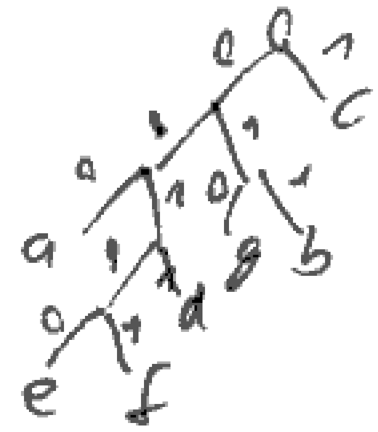
c a g b (ef) d
0,4 0,18 0,14 0,1 0,1 0,08

c a ((ef)d) g b
0,4 0,18 0,18 0,14 0,1

c gb a ((ef)d)
0,4 0,24 0,18 0,18

c a((ef)d) gb
0,4 0,36 0,24

(a((ef)d)gb) c
0,6 0,4



a : 000
b : 011
c : 1
d : 0011
e : 00100
f : 00101
g : 010

$H(S) = 2{,}427$
$\bar\lambda = 2{,}48$
$E = 0{,}979$
$R = 0{,}021$

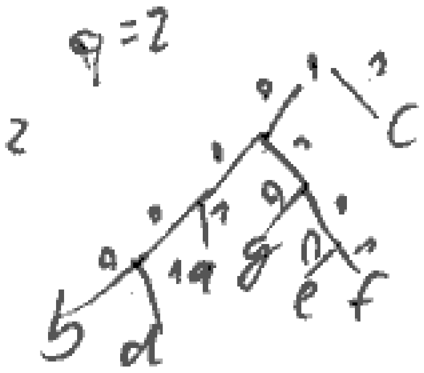
3) c a g ef b d $q=2$
0,4 0,18 0,14 0,1 0,1 0,08

c bd a g ef
0,4 0,18 0,18 0,14 0,1

c g(ef) bd a
0,4 0,24 0,18 0,18

(bd)a g(ef)
c
0,4 0,36 0,24

((bd)a)(g(ef)) c
0,6 0,4



(Tausch)

a : 001
b : 0000
c : 1
d : 0001
e : 0110
f : 0111
g : 010

$H(S) = 2{,}427$
$\bar\lambda = 2{,}48$
$E = 0{,}979$
$R = 0{,}021$

4) Bestimme $\frac{n}{q-1} = \frac{7}{2} = 3{,}5 = 3 + \text{Rest } 1$

c def a g b
0,4 0,18 0,18 0,14 0,1

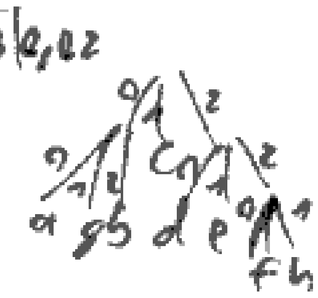
g agb | c | def
0,14 0,48 | 0,4 | 0,18

+h : 0,02 , f : 0,03 $\frac{n}{q-1} = 4 R 0$

⇒ c | a | g | b | d | e | f | h
0,4 | 0,18 | 0,14 | 0,1 | 0,08 | 0,05 | 0,03 | 0,02

⇒ c | a | g | b | d | e | fh
0,4 | 0,18 | 0,14 | 0,1 | 0,08 | 0,05 | 0,05

+ wie gerade ⇒



$w_1 = 1001$ Hamming-Abstand Hamming-Distanz
$w_2 = 1111$ $d(w_1, w_2) = 2$ $C = \{w_1, w_2, w_3\}$
$w_3 = 1101$ $d(w_1, w_3) = 1$ $d_0 = 1$. (Min. Abst.)
$d(w_2, w_3) = 1$

$[n, M, d_0, q]$: $[5, M, 2, 2]$ $C = \{0, 1\}$

$A = \{00000, 00011\}$

$M = q^{n+1-d_0} = 2^{5+1-2} = 16$ (Singelton m. Gleichheit)

$\frac{1}{2} \cdot 5 \nless 2 \Rightarrow ↯$

0000	0
0001	1
0010	1
0011	0
0100	1
0101	0
0110	0
0111	1
1000	1
1001	0
1010	0
1011	1
1100	0
1101	1
1110	1
1111	0

optimal $\Rightarrow$ Plotkin schr.: $q > 1$, $rn < d_0$, $r = \frac{q-1}{q}$

$M \overset{!}{=} \left\lfloor \frac{d_0}{d_0 - r \cdot n} \right\rfloor$

$\cancel{\frac{d_0}{d_0 - r n} = \frac{2}{2 - 5r}}$ $\cancel{r = \frac{1}{2}}$ $\cancel{\frac{2}{0,5}}$

perfekt $\Rightarrow$ Kugelschr.: $M = \frac{q^n}{\sum_{i=0}^{t} \binom{n}{i} (q-1)^i}$

$\Rightarrow ↯$

$t+1$

$[5, M, 3, 2] \Rightarrow M = \left\lfloor \frac{3}{3 - 2,5} \right\rfloor = 6$, $M = 2^{5+1-3} = 8$, $M = \frac{2^5}{\binom{5}{0} + \binom{5}{1} \cdot 2} = \frac{32}{11}$

wir möchten Eingabezeichen durch 9 Stellen codieren

1) (9,1) - Repitition - code $(2N+1, 1)$

2) benutze (3,1)-Rep. Code & codiere dann ~~a~~ nochmal mit (3,1)-Rep. code

1) $P_{Rest_1} = \sum_{i=0}^{N} \binom{2N+1}{i} \cdot (1-p_0)^i \cdot p_0^{2N+1-i}$

$P_{Rest_1} = \sum_{i=0}^{4} \binom{9}{i} (1-p_0)^i p_0^{9-i} \approx \binom{9}{4}(1-p_0)^4 p_0^5$

$\approx 126\, p_0^5 + \dots$

2) $P_{Rest_{3^1}} = \sum_{i=0}^{1} \binom{3}{i} (1-p_0)^i p_0^{3-i} \approx 3 p_0^2$

$\Rightarrow P_{Rest_2} \approx 3 \cdot (P_{Rest_{3^1}})^2 \approx 27\, p_0^4$

7

$$H = \left(\begin{array}{cccc|ccc} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$$

a) $$G = \left(\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right)$$

b) kodiere $z = (1,0,1,1)$

$$z \cdot G = (1,0,1,1) \cdot \left(\begin{array}{cccc|ccc} 1 & & & & 1 & 1 & 0 \\ & \ddots & & & 0 & 1 & 1 \\ & & \ddots & & 1 & 1 & 1 \\ 0 & & & 1 & 1 & 0 & 1 \end{array}\right) = (1,0,1,1 \mid 1,0,0)$$

c) dekodiere $y = (1,1,0,1,0,1,1)$

$$s^T = H y^T = \left(\begin{array}{cccc|ccc} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \Rightarrow s = (0,1,1)$$

$\Rightarrow (H \quad z)$

$$\hat{y} = y + e = (1101011) + (0100000) = (\underbrace{1001}\,001)$$

$F_2 : 0, 1 = -1$, $F_3 : 0, 1, 2 = -1$, -2

$\Rightarrow$ dec. $y^* = 1001$

zu 7.3: $q=4$, $m=2$:

Def: Ein (n,k)-BC über $GF(q)$, $q \geq 2$ heißt Hammingcode, falls

1) $n = \frac{q^m-1}{q-1}$, $m = n-k$, ~~2) Die Kontr.mat H ist $n-k \times n$-Matr, mit $\text{Rang}(H) = n-k$~~

3) $\min(\text{l.u. Spalten}(H))$ ist $\rho(H) \geq 2$

$$n = \frac{q^m-1}{q-1} = \frac{4^2-1}{3} = 5 \qquad k = 3 \quad \Rightarrow H : 2 \times 5\text{-Matrix}, \ \text{Rang}(H) = 2$$

je Zw-Paare lin. unabh.

$$\Rightarrow H = \begin{pmatrix} 1 & 0 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 & 2 \end{pmatrix} \Rightarrow \rho(H) = 2$$

	2	3
$\binom{1}{0}$	$\binom{2}{0}$	$\binom{3}{0}$
$\binom{0}{1}$	$\binom{0}{2}$	$\binom{0}{3}$
$\binom{1}{1}$	$\binom{2}{2}$	$\binom{3}{3}$
$\binom{2}{1}$	$\binom{0}{1}$	$\binom{1}{3}$
$\binom{3}{1}$	$\binom{2}{2}$	$\binom{1}{2}$

$\Rightarrow$ l.u. zur eind. Zuordnung d. Syndroms

2-Fehler-korr. Code: $\quad 2^n \geq 2^k \frac{n^2+n+2}{2}$, $k=2$ $\Rightarrow 2^n \geq 2(n^2+n+2)$

n	2^n	$2(n^2+n+2)$
3	8	16
4	16	⋮
5	32	⋮
7	128	116

$\Rightarrow (7,2)$-BC

$$\Rightarrow H = \left(\begin{array}{cc|ccccc} 1 & 1 & 1 & & & & 0 \\ 1 & 0 & & 1 & & & \\ 1 & 1 & & & 1 & & \\ 1 & 1 & & & & 1 & \\ 0 & 1 & 0 & & & & 1 \end{array}\right) \Rightarrow \rho(H) = 4$$

wählbar

y_1 (1 1 0 0 0 0 0)
y_2 (1 0 1 0 0 0 0)
y_3 (0 1 1 0 0 0 0)
...

$$H y_1^T = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad H y_2^T = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad H y_3^T = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

(1100000) (1010000) (0110000)

Zykl. Code C: 1. lin Code, 2. $a=(a_0 \dots a_{n-1}) \in C \Leftrightarrow b=(a_{n-1} a_0 a_1 \dots a_{n-2}) \in C$ (über $GF(q)$)

$a(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_{n-1} x^{n-1}$

Addition der Koeff mod(q), Multipl.: Polynommultipl mod(x^n-1)

$g(x)$: generatorpolynom mit $g(x) = c_0 + c_1 x + \dots + c_{k-1} x^{k-1} + x^k$, grad$(g) = k$

$$\Rightarrow G = \begin{pmatrix} c_0 & c_1 & \dots & c_{k-1} & 1 & 0 & \dots \\ 0 & c_0 & \dots & \dots & c_{k-1} & 1 & 0 \dots \\ 0 & 0 & c_0 & \dots & c_{k-1} & 1 & 0 \dots \\ \vdots & & \vdots & & & \ddots & 1\,0 \dots \end{pmatrix}, \quad h(x) = \frac{x^n-1}{g(x)} = h_0 + h_1 x + \dots + h_{m-1} x^{m-1} + x^m$$

$$H = \begin{pmatrix} 0 \dots 0\,1\,h_{m-1} \dots h_1 h_0 \\ \dots 0\,1\,h_{m-1} \dots h_1 h_0\, 0 \\ \dots 0\,1\,h_{m-1} \dots h_1 h_0\, 0\, 0 \\ \vdots \end{pmatrix}$$

Beispiel: $n=7$: $g(x) = 1+x^2+x^3+x^4$ $\mathbb{F}_2$ $\Rightarrow k=4$, $n-k=3$

$$\Rightarrow G = \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix} \Big| 3 = n-k, \quad h(x) = \frac{x^n-1}{g(x)}$$

$7 = n$

$(x^7-1) : (x^4+x^3+x^2+1) = x^3+x^2+1$

$x^7+x^6+x^5+x^3$

$\cancel{1}+x^6+x^5+x^3+1$

$+x^6+x^5+x^4+x^2$

$x^4+x^3+x^2+1$

$$\Rightarrow H = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Multipl.: $V(x) = Q(x)\, g(x)$, $Q = 100$, $Q(x) = 1+x$

$\Rightarrow V(x) = (1+x)(1+x^2+x^3+x^4)$

$= 1+x^2+x^3+x^4+x+x^3+x^4+x^5 = 1+x+x^2+x^5$

$= 1+x+x^2+x^5$

$\Rightarrow v = 1110010$

$g(x) = 1+x+x^4$ $a_1 = (1101000101001\,11)$

$(x^{14}+x^{13}+x^{12}+x^9+x^7+x^3+x+1) : (x^4+x+1) = x^{10}+x^? \dots$

$x^{14}+x^{11}+x^{10}$

$x^{13}+x^{12}+x^{11}+x^{10}+x^9+x^7+x^3+x+1$

$x^{13}+ \dots$

$(7,3)$-Kanalcode $\quad g(x) = 1 + x + x^2 + x^4$ über $\mathbb{F}_2$

Kanalcodewörter mit Div-Meth.?

$n = 7, \quad k = 4, \quad l = 7 - 4 = 3 \quad$, Kodiere 101

$Q(x) = 1 + x^2$

1) $\tilde{Q} = Q(x)\,x^4 = x^4 + x^6$

2) $\frac{\tilde{Q}(x)}{g(x)} = \underline{x^6 + x^4} : (x^4 + x^2 + x - 1) = x^2 + R : \underbrace{x^3 + x^2}_{r}$

3) $Q^*(x) = \tilde{Q} + r = x^2 + x^3 + x^4 + x^6$

0011101

$m(x) = 1 + x^3 + x^4$ über $\mathbb{F}_2$, α ist Ns, primitiv? (Erweitg. konst.)

$|GF(2^4)| = 16$ $\qquad 1 + \alpha^3 + \alpha^4 = 0$

$0,\ \alpha^0 = 1,\ \alpha^1 = \alpha,\ \alpha^2 = \alpha^2,\ \alpha^3 = \alpha^3,\ \alpha^4 = \alpha^3 + 1$

$\alpha^5 = \alpha\alpha^4 = \alpha\alpha^3 + \alpha = \alpha^3 + \alpha + 1,\ \alpha^6 = \alpha^2\alpha^3 + \alpha^2 = \alpha^3 + \alpha^2 + \alpha + 1$

$\alpha^7 = \alpha^3\alpha^4 = \alpha^3\alpha^3 + \alpha^3 = \alpha^3 + \alpha^2 + \alpha + 1 + \alpha^3 = \alpha^2 + \alpha + 1$

$\alpha^8 = \alpha\alpha^7 = \alpha^3 + \alpha^2 + \alpha,\ \alpha^9 = \alpha\alpha^8 = \alpha^4 + \alpha^3 + \alpha^2 = \alpha^2 + 1$

$\alpha^{10} = \alpha^3 + \alpha,\ \alpha^{11} = \alpha\alpha^{10} = \alpha^4 + \alpha^2 = \alpha^3 + \alpha^2 + 1$

$\alpha^{12} = \alpha\alpha^{11} = \alpha^4 + \alpha^3 + \alpha = \alpha + 1$

$\alpha^{13} = \alpha\alpha^{12} = \alpha^2 + \alpha,\ \alpha^{14} = \alpha\alpha^{13} = \alpha^3 + \alpha^2$

$\alpha^{15} = \alpha^4 + \alpha^3 = 1 = \alpha^0 \Rightarrow 16$ El.

Minimalpolynome: $\alpha^1, \alpha^2, \alpha^4, \alpha^8, \alpha^{16} = \alpha$, $\alpha^0: m_0(x) = x + 1$

$m_1(x) = (x-\alpha)(x-\alpha^2)(x-\alpha^4)(x-\alpha^8)$

$\alpha^3, \alpha^6, \alpha^{12}, \alpha^{24} = \alpha^9, \alpha^{18} = \alpha^3 \Rightarrow m_2(x) = (x-\alpha^3)(x-\alpha^6)(x-\alpha^{12})(x-\alpha^9)$

$\alpha^5, \alpha^{10}, \alpha^{20} = \alpha^5 \Rightarrow m_3(x) = (x-\alpha^5)(x-\alpha^{10})$

$\alpha^7, \alpha^{14}, \alpha^{28} = \alpha^{13}, \alpha^{26} = \alpha^{11}, \alpha^{22} = \alpha^7 \Rightarrow m_4(x) = (x-\alpha^7)(x-\alpha^{14})(x-\alpha^{13})(x-\alpha^{11})$

$m_3(x) = x^2 + x\underbrace{(\alpha^5 + \alpha^{10})}_{\alpha^3+\alpha+1+\alpha^3+\alpha\,=\,1} + \underbrace{\alpha^{15}}_{1} = x^2 + x + 1$

Sardinas Patterson:

$S_0 = C$

$\hat{S}_i = \{w \in B^+ \mid \exists w_1 \in C$ und $\exists w_2 \in S_{i-1}, w_2 = w_1 w\}$

$\tilde{S}_i = \{w \in B^+ \mid \exists w_1 \in S_{i-1}$ und $\exists w_2 \in C, w_2 = w_1 w\}$

$S_i = \hat{S}_i \cup \tilde{S}_i$, $S = \bigcup_{i \geq 0} S_i$, C eind. dek. $\Leftrightarrow C \cap S = \emptyset$

$C = \{01, 11, 012, 020, 100, 0021, 0122, 1110\}$

$S_0 = C$

$\hat{S}_1 = \tilde{S}_1 = \{2, 22, 10\}$

$\hat{S}_2 = \{\}$ $\qquad \tilde{S}_2 = \{0\} = S_2$

$\hat{S}_3 = \{\}$ $\qquad \tilde{S}_3 = \{1, 12, 20, 021, 122\} = S_3$

$\hat{S}_4 = \{\}$ $\qquad \tilde{S}_4 = \{00, 110\} = S_4$

$\hat{S}_5 = \{0\}$ $\qquad \tilde{S}_5 = \{00, 110, 1, 21\} \Rightarrow S_5 = \{0, 00, 1, 110, 21\}$

$\hat{S}_6 = \{0\}$ $\qquad \tilde{S}_6 = \{00, 1, 110, 12, 20, 21, 122\} \Rightarrow S_6 = \{0, 00 \ldots\}$

$\hat{S}_7 = \{0\}$ $\qquad \tilde{S}_7 = \{1, 12, 20, 021, 122, 1, 00, 110, 21\} \Rightarrow S_7 = S_6$

$S = \{2, 22, 0, 1, 12, 20, 021, 122, 00, 110, 21\}$, $S \cap C = \emptyset$