

Problem 1.1

$$m_{\Delta^{++}} = 1232 \frac{\text{MeV}}{c^2}, \quad m_{\pi} = 139,570 \frac{\text{MeV}}{c^2}, \quad m_p = 938,272 \frac{\text{MeV}}{c^2}$$

particle at rest: $E_G = m_{\Delta^{++}} c^2$, $E_{\pi} + E_p = E_G$, $|\vec{p}_{\pi}| = |\vec{p}_p|$

$$\Rightarrow 0 = |\vec{p}_{\pi}| - |\vec{p}_p| = \frac{1}{c} (\sqrt{E_{\pi}^2 - m_{\pi}^2 c^4} - \sqrt{E_p^2 - m_p^2 c^4})$$

$$= \frac{1}{c} (\sqrt{E_{\pi}^2 - m_{\pi}^2 c^4} - \sqrt{(E_G - E_{\pi})^2 - m_p^2 c^4})$$

$$\Leftrightarrow E_{\pi}^2 - m_{\pi}^2 c^4 = E_G^2 - 2 E_G E_{\pi} + E_{\pi}^2 - m_p^2 c^4$$

$$\Leftrightarrow 2 E_G E_{\pi} = E_G^2 + m_{\pi}^2 c^4 - m_p^2 c^4$$

$$\Leftrightarrow E_{\pi} = \frac{1}{2} \left(E_G + \frac{m_{\pi}^2 c^4}{m_{\Delta^{++}}} - \frac{m_p^2 c^4}{m_{\Delta^{++}}} \right)$$

$$= \frac{1}{2} \left(1232 \text{ MeV} + \frac{(139,57)^2}{1232} \text{ MeV} - \frac{(938,272)^2}{1232} \text{ MeV} \right)$$

$$= 266,619 \text{ MeV}$$

$$E_p = \frac{1}{2} \left(E_G + \frac{m_p^2 c^4}{m_{\Delta^{++}}} - \frac{m_{\pi}^2 c^4}{m_{\Delta^{++}}} \right)$$

$$= 965,381 \text{ MeV}$$

Relativist.!

$$|\vec{p}_{\pi}| = \frac{1}{c} \sqrt{E_{\pi}^2 - m_{\pi}^2 c^4} = 227,169 \frac{\text{MeV}}{c} = |\vec{p}_p|$$

$$E_{\text{kin}, \pi} = \frac{|\vec{p}_{\pi}|^2}{2 m_{\pi}} = 184,875 \text{ MeV} \quad f$$

$$E_{\text{kin}, p} = \frac{|\vec{p}_p|^2}{2 m_p} = 27,5005 \text{ MeV} \quad (v)$$

$$\beta_{\pi} = \frac{|\vec{p}_{\pi}|}{E_{\pi}} = \frac{227,169 \text{ MeV}}{266,619 \text{ MeV}} = 0,852037$$

$$\beta_p = \frac{|\vec{p}_p|}{E_p} = \frac{227,169 \text{ MeV}}{965,381 \text{ MeV}} = 0,235316$$

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Problem 1.2

Sch w.

a) $p + p \rightarrow \pi^+ + \pi^0$	X	electrical charge violated ($1+1 \neq 1+0$)
b) $\eta \rightarrow \gamma + \gamma$	✓	EM-interaction. (Parity: $(-1) \rightarrow (-1) \cdot (-1) \cdot (-1)^L = -1$)
c) $\Sigma^0 \rightarrow \Lambda + \pi^0$	X	Parity violated ($(+1) \rightarrow (+1) \cdot (-1)$)
d) $\Sigma^- \rightarrow n + \pi^-$	✓	weak interaction: ($dds \rightarrow udd + \bar{u}d$)
e) $e^+ + e^- \rightarrow \mu^+ + \mu^-$	✓	EM-interaction, but much Energy needed!
f) $\mu^- \rightarrow e^- + \bar{\nu}_e$	X	Lepton number violated ($L: 1 \rightarrow 1 + 1$)
g) $\Delta^+ \rightarrow p + \pi^0$	✓	strong interaction ($J: \frac{3}{2} \rightarrow \frac{1}{2} + 0 + 1$ ($\in L$))
h) $\bar{\nu}_e + p \rightarrow n + e^+$	✓	weak interaction ($und \rightarrow udd$)
i) $e^- + p \rightarrow \nu_e + \pi^0$	X	Baryon number violated ($B: 0 + 1 \rightarrow 0 + 0$)
f j) $p + p \rightarrow \Sigma^+ + n + K^0 + \pi^+ + \pi^0$	X	Parity violated (Parity: $(+1)^2 \rightarrow (+1)^2 (-1)^3$)
k) $p \rightarrow e^+ + \gamma$	X	Baryon number violated ($B: 1 \rightarrow 0 + 0$)
l) $p + p \rightarrow p + p + p + \bar{p}$	✓	strong interaction (+Energy)
m) $n + \bar{n} \rightarrow \pi^+ + \pi^- + \pi^0$	✓	strong interaction
n) $\pi^+ + n \rightarrow \pi^- + p$	X	electrical charge violated ($+1 + 0 \rightarrow -1 + 1$)
o) $K^- \rightarrow \pi^- + \pi^0$	✓	weak interaction ($s\bar{u} \rightarrow d\bar{u} + \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$)
p) $\Sigma^+ + n \rightarrow \Sigma^- + p$	X	electrical charge violated ($+1 + 0 \rightarrow -1 + 1$)
q) $\Sigma^0 \rightarrow \Lambda + \gamma$	✓	EM-interaction (Parity: $(+1) \rightarrow (+1)(-1)(-1)^L = +1$)
r) $\Xi^- \rightarrow \Lambda + \pi^-$	✓	weak interaction ($dss \rightarrow uds + d\bar{u}$)
s) $\Xi^0 \rightarrow p + \pi^-$	X	$\Delta S = 2 \Rightarrow$ highly suppressed (10^{-8}) weak l.
t) $\pi^- + p \rightarrow \Lambda + K^0$	✓	strong interaction ($d\bar{u} + uud \rightarrow uds + d\bar{s}$)
u) $\pi^0 \rightarrow \gamma + \gamma$	✓	EM-interaction (Parity: $(-1) \rightarrow (-1)(-1)(-1)^L = -1$)
v) $\Sigma^- \rightarrow n + e^- + \bar{\nu}_e$	✓	weak interaction ($dds \rightarrow udd$)
f w) $n \rightarrow p + \mu^- + \bar{\nu}_\mu$	✓	weak interaction (high Energy needed)
x) $\Delta^+ \rightarrow p$	X	angular momentum violated ($J: \frac{3}{2} \rightarrow \frac{1}{2}$)

Problem 1.3

a) $\pi^-: |1, -1\rangle, p: |\frac{1}{2}, \frac{1}{2}\rangle$

$$|\pi^- p\rangle \hat{=} |1, -1\rangle |\frac{1}{2}, \frac{1}{2}\rangle \hat{=} \sqrt{\frac{1}{3}} |\frac{3}{2}, -\frac{1}{2}\rangle - \sqrt{\frac{2}{3}} |\frac{1}{2}, -\frac{1}{2}\rangle$$

$$K^0: \langle \frac{1}{2}, -\frac{1}{2} |, \Sigma^0: \langle 1, 0 |$$

$$\langle \Sigma^0 K^0 | \hat{=} \langle 1, 0 | \langle \frac{1}{2}, -\frac{1}{2} | \hat{=} \sqrt{\frac{2}{3}} \langle \frac{3}{2}, -\frac{1}{2} | + \sqrt{\frac{1}{3}} \langle \frac{1}{2}, -\frac{1}{2} |$$

$$\Rightarrow \langle \Sigma^0 K^0 | T | \pi^- p \rangle = \frac{\sqrt{2}}{3} \langle \frac{3}{2}, -\frac{1}{2} | T | \frac{3}{2}, -\frac{1}{2} \rangle - \frac{\sqrt{2}}{3} \langle \frac{1}{2}, -\frac{1}{2} | T | \frac{1}{2}, -\frac{1}{2} \rangle$$

$$= \frac{\sqrt{2}}{3} T_{3/2} - \frac{\sqrt{2}}{3} T_{1/2} = M_a$$

b) $|\pi^- p\rangle \hat{=} \sqrt{\frac{1}{3}} |\frac{3}{2}, -\frac{1}{2}\rangle - \sqrt{\frac{2}{3}} |\frac{1}{2}, -\frac{1}{2}\rangle$

$$K^+: \langle \frac{1}{2}, \frac{1}{2} |, \Sigma^-: \langle 1, -1 |$$

$$\langle \Sigma^- K^+ | \hat{=} \langle 1, -1 | \langle \frac{1}{2}, \frac{1}{2} | \hat{=} \sqrt{\frac{1}{3}} \langle \frac{3}{2}, -\frac{1}{2} | - \sqrt{\frac{2}{3}} \langle \frac{1}{2}, -\frac{1}{2} |$$

$$\Rightarrow \langle \Sigma^- K^+ | T | \pi^- p \rangle = \frac{1}{3} \langle \frac{3}{2}, -\frac{1}{2} | T | \frac{3}{2}, -\frac{1}{2} \rangle + \frac{2}{3} \langle \frac{1}{2}, -\frac{1}{2} | T | \frac{1}{2}, -\frac{1}{2} \rangle$$

$$= \frac{1}{3} T_{3/2} + \frac{2}{3} T_{1/2} = M_b$$

c) $\pi^+: |1, 1\rangle, p: |\frac{1}{2}, \frac{1}{2}\rangle, K^+: \langle \frac{1}{2}, \frac{1}{2} |, \Sigma^+: \langle 1, 1 |$

$$|\pi^+ p\rangle \hat{=} |1, 1\rangle |\frac{1}{2}, \frac{1}{2}\rangle \hat{=} |\frac{3}{2}, \frac{3}{2}\rangle$$

$$\langle \Sigma^+ K^+ | \hat{=} \langle 1, 1 | \langle \frac{1}{2}, \frac{1}{2} | \hat{=} \langle \frac{3}{2}, \frac{3}{2} |$$

$$\Rightarrow \langle \Sigma^+ K^+ | T | \pi^+ p \rangle = \langle \frac{3}{2}, \frac{3}{2} | T | \frac{3}{2}, \frac{3}{2} \rangle$$

$$= T_{3/2} = M_c$$

$\Rightarrow$ branching ratios: $\sigma_i \sim M_i^2$ a) b) c)

$$l = \frac{3}{2} \text{ dominant} \Rightarrow T_{3/2} = 0 \quad : \quad \frac{2}{9} : \frac{1}{9} : 1$$

$$l = \frac{1}{2} \text{ dominant} \Rightarrow T_{1/2} = 0 \quad : \quad \frac{2}{9} : \frac{4}{9} : 0$$

$$\hat{=}$$

$$l = \frac{3}{2} : \quad a) \ b) \ c)$$

$$2 : 1 : 9$$

$$l = \frac{1}{2} : \quad 1 : 2 : 0$$

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