Hadron and Nuclear Physics

Exercise sheet 1

Winter term 2013/14

Hand-in: In the exercise group on Nov 5th 2013

Problem 1.1: Decay of the Δ-Resonance (12 points)

(a) In pion-nucleon scattering at a center-of-mass energy of 1232 MeV, the short-lived Δ-resonance is formed which was discovered in 1951 by Fermi and coworkers. The Δ^{++} decays according to

$$\Delta^{++} \rightarrow p + \pi^+,$$

where $m_\pi = 139.570\,\mathrm{MeV/c^2}$.
Calculate, by applying relativistic kinematics to the decay of the Δ-resonance at rest, the energies and momenta of the final state particles as well as their kinetic energies and "velocities" β_p and β_π.

Problem 1.2: Conservation Laws (12 points)

(a) Examine the following processes, and state for each one wether it is *possible* or *impossible*, according to the Standard Model. In the former case, state which interaction is responsible - strong, electromagnetic, or weak; in the latter case, cite a conservation law that prevents it from occuring.

a) $p + \bar{p} \rightarrow \pi^+ + \pi^0$
b) $\eta \rightarrow \gamma + \gamma$
c) $\Sigma^0 \rightarrow \Lambda + \pi^0$
d) $\Sigma^- \rightarrow n + \pi^-$
e) $e^+ + e^- \rightarrow \mu^+ + \mu^-$
f) $\mu^- \rightarrow e^- + \bar{\nu}_e$
g) $\Delta^+ \rightarrow p + \pi^0$
h) $\bar{\nu}_e + p \rightarrow n + e^+$
i) $e^- + p \rightarrow \nu_e + \pi^0$
j) $p + p \rightarrow \Sigma^+ + n + K^0 + \pi^+ + \pi^0$
k) $p \rightarrow e^+ + \gamma$
l) $p + p \rightarrow p + p + p + \bar{p}$
m) $n + \bar{n} \rightarrow \pi^+ + \pi^- + \pi^0$
n) $\pi^+ + n \rightarrow \pi^- + p$
o) $K^- \rightarrow \pi^- + \pi^0$
p) $\Sigma^+ + n \rightarrow \Sigma^- + p$
q) $\Sigma^0 \rightarrow \Lambda + \gamma$
r) $\Xi^- \rightarrow \Lambda + \pi^-$
s) $\Xi^0 \rightarrow p + \pi^-$
t) $\pi^- + p \rightarrow \Lambda + K^0$
u) $\pi^0 \rightarrow \gamma\gamma$
v) $\Sigma^- \rightarrow n + e^- + \bar{\nu}_e$
w) $n \rightarrow p + \mu^- + \bar{\nu}_\mu$
x) $\Delta^+ \rightarrow p$

Problem 1.3: Isospin (12 points)

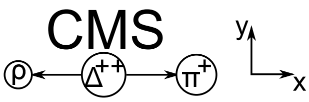
(a) Find the ratio of the cross sections of the following reactions, assuming the CM energy is such that $I = \frac{3}{2}$ dominates:

a) $\pi^- + p \rightarrow K^0 + \Sigma^0$
b) $\pi^- + p \rightarrow K^+ + \Sigma^-$
c) $\pi^+ + p \rightarrow K^+ + \Sigma^+$.

What if the energy is such that the $I = \frac{1}{2}$ channel dominates?

1 Übung: Besprechung

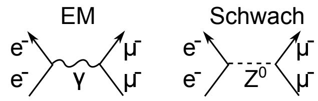
1.1 Aufgabe



$\hbar = c = 1$
$p_p = (E_p, p, 0, 0)$
$s = (p_p + p_\pi)^2 = (E_p + E_\pi)^2 \quad \Rightarrow \quad \sqrt{s} = E_p + E_\pi$
$p_p^2 = m_p^2 = E_p^2 - p^2 \;, \quad p_\pi^2 = m_\pi^2 = E_\pi^2 - p^2$

Impulserhaltung:
$p^2 = p^2 \quad \Leftrightarrow \quad E_p^2 - m_p^2 = E_\pi^2 - m_\pi^2$
$\Leftrightarrow \quad E_p^2 - m_p^2 = E_\pi^2 - m_\pi^2 = (E_p - m_p)(E_p + m_p) = \sqrt{s}(E_p - m_p)$
$E_p = \frac{1}{2}(\sqrt{s} + \frac{m_p^2 - m_\pi^2}{\sqrt{s}})$
$E_p = \frac{1}{2}(\sqrt{s} + \frac{m_\pi^2 - m_p^2}{\sqrt{s}})$
$T_p = E_p - m_p \;, \quad T_\pi = E_\pi - m_\pi$
$p = \sqrt{E_p^2 - m_p^2} = p_\pi$
$\beta_p = \frac{p}{E_p} \;, \quad \beta_\pi = \frac{p}{E_\pi}$

1.2 Aufgabe: Teile

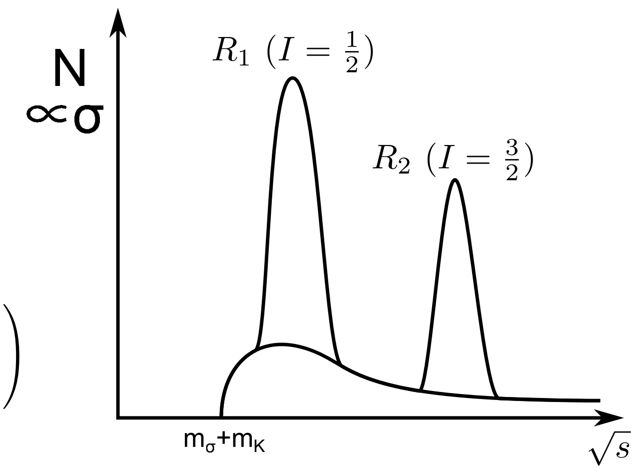


e) $e^+ + e^- \to \mu^+ + \mu^-$:

j) $p + p \to \Sigma^+ + n + K^0 + \pi^+ + \pi^0$: stark, Parität: je 2 Teilchen können l beitragen ⇒ erhalten.

1.3 Aufgabe: Besprechung

Toy-Modell:



$\begin{pmatrix} p \\ n \end{pmatrix} \quad \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \quad \begin{pmatrix} \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{pmatrix} \quad \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$

$4\pi^- + p \to K^0 + \Sigma^0$:
$|1, -1 > |\frac{1}{2}, \frac{1}{2} > \to |\frac{1}{2}, -\frac{1}{2} > 1, 0| >$
$\Leftrightarrow \sqrt{\frac{1}{3}}|\frac{3}{2}, -\frac{1}{2} > - \sqrt{\frac{2}{3}}|\frac{1}{2}, -\frac{1}{2} > \to \sqrt{\frac{2}{3}}|\frac{3}{2}, -\frac{1}{2} > + \sqrt{\frac{1}{3}}|\frac{1}{2}, -\frac{1}{2} >$

$M =< I, M|T|I', M' >$
$M_1 \quad =< \pi^- p|T|K^0\Sigma^0 >$
$\quad = (\sqrt{\frac{1}{3}} < \frac{3}{2}, -\frac{1}{2}| - \sqrt{\frac{2}{3}} < \frac{1}{2}, -\frac{1}{2}|)T(\sqrt{\frac{2}{3}}|\frac{3}{2}, -\frac{1}{2} > + \sqrt{\frac{1}{3}}|\frac{1}{2}, -\frac{1}{2} >)$
$\quad = \frac{\sqrt{2}}{3}T_{3/2} - \frac{\sqrt{2}}{3}T_{1/2}$
$M_2 =< \pi^- p|T|K^+\Sigma^- >= \frac{1}{3}T_{3/2} + \frac{2}{3}T_{1/2}$
$M_3 =< \pi^+ p|T|K^+\Sigma + >= T_{3/2}$

$\sigma \propto M^2$
$I = \frac{3}{2} \quad \Rightarrow T_{1/2} = 0 \quad \Rightarrow \quad \frac{2}{9} \;:\; \frac{1}{9} \;:\; 1 \quad \Leftrightarrow \quad 2 \;:\; 1 \;:\; 9$
$I = \frac{1}{2} \quad \Rightarrow T_{3/2} = 0 \quad \Rightarrow \quad \frac{2}{9} \;:\; \frac{4}{9} \;:\; 0 \quad \Leftrightarrow \quad 1 \;:\; 2 \;:\; 0$