

1 Präsenzaufgaben, 5.11.13

Isospin lässt sich zurückführen auf Quarks:

$$\begin{pmatrix} p \\ n \end{pmatrix} \rightarrow \begin{pmatrix} u \\ d \end{pmatrix} \Rightarrow \begin{aligned} u &= |\frac{1}{2}, \frac{1}{2}\rangle \\ d &= |\frac{1}{2}, -\frac{1}{2}\rangle \end{aligned}$$

Für Antiteilchen:

$$\begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} \Rightarrow \begin{aligned} \bar{d} &= |-\frac{1}{2}, \frac{1}{2}\rangle \\ \bar{u} &= |\frac{1}{2}, -\frac{1}{2}\rangle \end{aligned}$$

Nur u&d Quarks: SU(2)

$$2 \otimes 2 = 3 \oplus 1 \Rightarrow N = 2^2$$

$$\Rightarrow \begin{cases} \text{symmetrisch} & \begin{cases} |1, 1\rangle & = |\pi^+\rangle & = -|u\bar{d}\rangle \\ |1, 0\rangle & = |\pi^0\rangle & = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle - |d\bar{d}\rangle) \\ |1, -1\rangle & = |\pi^-\rangle & = |\bar{u}d\rangle \end{cases} \\ \text{antisymmetrisch} & |0, 0\rangle = |\eta''\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle) \end{cases}$$

u&d&s Quarks: SU(3)

Erwartet: $N = 3^2 = 9$

$3 \otimes \bar{3} = 8 \oplus 1$

Flavour Wellenfunktion (nicht symmetrisch):

$$\begin{aligned} K^+ &= n\bar{s} \\ K^0 &= d\bar{s} \\ \bar{K}^0 &= -s\bar{d} \\ K^- &= -s\bar{u} \end{aligned}$$

Oktett:

$$\begin{aligned} |\pi^0\rangle &= \frac{1}{\sqrt{2}}(|u\bar{u}\rangle - |d\bar{d}\rangle), \\ |\eta\rangle &= \frac{1}{\sqrt{6}}(|u\bar{u}\rangle - |d\bar{d}\rangle - 2|s\bar{s}\rangle), \\ |\eta'\rangle &= \frac{1}{\sqrt{3}}(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle) \end{aligned}$$

Quarks sind Fermionen

Spin = $\frac{1}{2}$, Wellenf. ϕ antisymmetrisch

$\phi = \phi_{ort} * \phi_{flavour} * \phi_{spin} * \phi_{color}$

Alle physikalischen Zustände sind Farbsingulets

Für Mesonen: $|\phi_{color}\rangle = \frac{1}{\sqrt{3}}(|r\bar{r}\rangle + |g\bar{g}\rangle + |b\bar{b}\rangle)$ (antisym.)

Beispiel: π^-

$$\begin{aligned} \phi &= \phi_{ort} * \phi_{flavour} * \phi_{spin} * \phi_{color} \\ \ominus &= \oplus * \oplus * \overset{!}{\oplus} * \ominus \\ |\pi^-\rangle_{SF} &= |\bar{u}d\rangle * (\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)) = \frac{1}{\sqrt{2}}(|\bar{u}_\uparrow d_\downarrow\rangle + |\bar{u}_\downarrow d_\uparrow\rangle) \end{aligned}$$