1 Präsenzaufgaben, 5.11.13

Isospin lässt sich zurückführen auf Quarks:

$$\begin{pmatrix} p \\ n \end{pmatrix} \rightarrow \begin{pmatrix} u \\ d \end{pmatrix} \quad \Rightarrow \quad \begin{aligned} u &= |\tfrac{1}{2}, \tfrac{1}{2} > \\ d &= |\tfrac{1}{2}, -\tfrac{1}{2} > \end{aligned}$$

Für Antiteilchen:

$$\begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} \quad \Rightarrow \quad \begin{aligned} \bar{d} &= -|\tfrac{1}{2}, \tfrac{1}{2} > \\ \bar{u} &= |\tfrac{1}{2}, -\tfrac{1}{2} > \end{aligned}$$

Nur u&d Quarks: SU(2)

$2 \otimes 2 = 3 \oplus 1 \quad \Rightarrow N = 2^2$

$$\Rightarrow \begin{array}{llll} \text{symmetrisch} \left\{ \begin{array}{l} \\ \\ \\ \end{array} \right. & \begin{array}{l} |1,1> \\ |1,0> \\ |1,-1> \end{array} & \begin{array}{l} = |\pi^+> \\ = |\pi^0> \\ = |\pi^-> \end{array} & \begin{array}{l} = -|u\bar{d}> \\ = \frac{1}{\sqrt{2}}(|u\bar{u}> -|d\bar{d}>) \\ = |\bar{u}d> \end{array} \\ \text{antisymmetrisch} & |0,0> & = |''\eta''> & = \frac{1}{\sqrt{2}}(|u\bar{u}> +|d\bar{d}>) \end{array}$$

u&d&s Quarks: SU(3)

Erwartet: $N = 3^2 = 9$

$3 \otimes \bar{3} = 8 \oplus 1$

Flavour Wellenfuncktion (nicht symmetrisch): $\begin{aligned} K^+ &= n\bar{s} \\ K^0 &= d\bar{s} \\ \bar{K}^0 &= -s\bar{d} \\ K^- &= -s\bar{u} \end{aligned}$

Oktett:

$|\pi^0 >= \frac{1}{\sqrt{2}}(|u\bar{u}> -|d\bar{d}>)$,

$|\eta >= \frac{1}{\sqrt{6}}(|u\bar{u}> -|d\bar{d}> -2|s\bar{s}>)$,

$|\eta' >= \frac{1}{\sqrt{3}}(|u\bar{u}> +|d\bar{d}> +|s\bar{s}>)$

Quarks sind Fermionen

Spin=$\frac{1}{2}$, Wellenfu. ϕ antisymmetrisch

$\phi = \phi_{ort} * \phi_{flavour} * \phi_{spin} * \phi_{color}$

Alle physikalischen Zustände sind Farbsinguletts

Für Mesonen: $|\phi_{color} >= \frac{1}{\sqrt{3}}(|r\bar{r}> +|g\bar{g}> +|b\bar{b}>)$ (antisym.)

Beispiel: π^-

$$\begin{array}{ccccccccc} \phi & = & \phi_{ort} & * & \phi_{flavour} & * & \phi_{spin} & * & \phi_{color} \\ \ominus & = & \oplus & * & \oplus & * & \stackrel{!}{=} \oplus & * & \ominus \end{array}$$

$|\pi^- >_{SF} = |\bar{u}d> *(\frac{1}{\sqrt{2}}(|\uparrow\downarrow> +|\downarrow\uparrow>)) = \frac{1}{\sqrt{2}}(|\bar{u}_\uparrow d_\downarrow> +|\bar{u}_\downarrow d_\uparrow>)$