Höhere Hadronenphysik

Julian Bergmann
Christian Lang

Problem 2.1

$$m_q = \frac{1}{3} m_N \quad , \quad \mu_q = \frac{Q_q \hbar}{2 m_q c}$$

$$\mu_{3q} = 2 \langle q_1 q_2 q_3 | \mu_1 S_{z1} + \mu_2 S_{z2} + \mu_3 S_{z3} | q_1 q_2 q_3 \rangle$$

$$|N_\uparrow\rangle = \sqrt{\tfrac{2}{9}} \left(|1_\uparrow 2_\uparrow 3_\downarrow\rangle + |3_\downarrow 1_\uparrow 2_\uparrow\rangle + |2_\uparrow 3_\downarrow 1_\uparrow\rangle \right) -$$
$$\sqrt{\tfrac{1}{18}} \left(|1_\downarrow 2_\uparrow 3_\uparrow\rangle + |3_\uparrow 1_\downarrow 2_\uparrow\rangle + |2_\uparrow 3_\uparrow 1_\downarrow\rangle + |1_\uparrow 2_\downarrow 3_\uparrow\rangle + |3_\uparrow 1_\uparrow 2_\downarrow\rangle + |2_\uparrow 3_\uparrow 1_\downarrow\rangle \right)$$

$$p: |123\rangle = |uud\rangle \quad , \quad n: |123\rangle = |ddu\rangle$$

$$\mu_u = \frac{2}{3} e \frac{\hbar}{2 m_q c^2} = 2 \frac{e\hbar}{6 m_q c^2} =: 2 \mu_N$$

$$\mu_d = -\frac{1}{3} e \frac{\hbar}{2 m_q c^2} = - \frac{e\hbar}{6 m c^2} =: -\mu_N$$

$$|p_\uparrow\rangle = \sqrt{\tfrac{2}{9}} \left(|u_\uparrow u_\uparrow d_\downarrow\rangle + |d_\downarrow u_\uparrow u_\uparrow\rangle + |u_\uparrow d_\downarrow u_\uparrow\rangle \right) -$$
$$\sqrt{\tfrac{1}{18}} \left(|u_\downarrow u_\uparrow d_\uparrow\rangle + |d_\uparrow u_\downarrow u_\uparrow\rangle + |u_\uparrow d_\uparrow u_\downarrow\rangle + |u_\uparrow u_\downarrow d_\uparrow\rangle + |d_\uparrow u_\uparrow u_\downarrow\rangle + |u_\uparrow d_\uparrow u_\downarrow\rangle \right)$$

$$\langle u_\uparrow u_\uparrow d_\downarrow | T_\mu | u_\uparrow u_\uparrow d_\downarrow \rangle = 2 \langle uud | 2\mu_N \cdot \tfrac{1}{2} + 2\mu_N \cdot \tfrac{1}{2} + \mu_N \cdot \tfrac{1}{2} | uud \rangle = 5 \mu_N$$

$$\langle u_\downarrow u_\uparrow d_\uparrow | T_\mu | u_\uparrow u_\uparrow d_\uparrow \rangle = 2 \langle uud | 2\mu_N \cdot \tfrac{1}{2} - 2\mu_N \cdot \tfrac{1}{2} - \mu_N \cdot \tfrac{1}{2} | uud \rangle = -\mu_N$$

$$\mu_p = \langle p_\uparrow | p_\uparrow \rangle = \sqrt{\tfrac{2}{9}}^2 \cdot 3 \cdot \langle u_\uparrow u_\uparrow d_\downarrow | T_\mu | u_\uparrow u_\uparrow d_\downarrow \rangle + \sqrt{\tfrac{1}{18}} \cdot 6 \cdot \langle u_\downarrow u_\uparrow d_\uparrow | T_\mu | u_\downarrow u_\uparrow d_\uparrow \rangle$$
$$= \frac{2}{9} \cdot 3 \cdot 5\mu_N + \frac{1}{18} \cdot 6 \cdot (-\mu_N) = 3 \mu_N$$

$$\langle d_\uparrow d_\uparrow u_\downarrow | T_\mu | d_\uparrow d_\uparrow u_\downarrow \rangle = 2 \langle ddu | -\mu_N \cdot \tfrac{1}{2} + (-\mu_N) \cdot \tfrac{1}{2} - \frac{2\mu_N}{2} | ddu \rangle = -4 \mu_N$$

$$\langle d_\downarrow d_\uparrow u_\uparrow | T_\mu | d_\uparrow d_\downarrow u_\uparrow \rangle = 2 \langle ddu | \mu_N \cdot \tfrac{1}{2} + (-\mu_N) \cdot \tfrac{1}{2} + \frac{2\mu_N}{2} | ddu \rangle = 2 \mu_N$$

$$\mu_n = \sqrt{\tfrac{2}{9}}^2 \cdot 3 \cdot \langle d_\uparrow d_\uparrow u_\downarrow | T_\mu | d_\uparrow d_\uparrow u_\downarrow \rangle + \sqrt{\tfrac{1}{18}} \cdot 6 \cdot \langle d_\uparrow d_\downarrow u_\uparrow | T_\mu | d_\uparrow d_\downarrow u_\uparrow \rangle$$
$$= \frac{2}{9} \cdot 3 \cdot (-4\mu_N) + \frac{1}{18} \cdot 6 \cdot 2 \mu_N = -2 \mu_N$$

$\Rightarrow$

magnetic moments	calculated	measured
μ_p	$3\mu_N$	$2{,}793 \mu_N$
μ_n	$-2\mu_N$	$-1{,}913 \mu_N$

higher hadron and Nuclear Physics

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Problem 2.3

Due to energy conservation the highest amount of energy available after collision is the cinetic energy of our fired proton (300 MeV) plus the resting masses of our particles.
However, as the initial particles (p+p) are found again after the collision we measured, only the initial cinetic energy is available to distribute to the two particles momentum after collision and to particle production.
Therefore the difference between the initial cinetic energy of 300 MeV and the sum of the finally measured cinetic energies of both protons is the available energy for particle production if both measured cinetic energies are equal.
In case they are not equal, energy has been released by one in form of a photon. This is the reason why in our calculation the upper cinetic energy is used to calculate the available energy for particle production while the difference is assumed to be the result of photon production.

T_3(MeV)	T_4(MeV)	reaction	particle
80,9	80,9	particle product.	300 MeV → 161,8 MeV : 138,2 MeV $\hat{=} \pi^0$
70	70	particle prod.	160 MeV $\hat{=} \pi^0$
50,2	50,2	particle prod.	199,6 $\hat{=} \pi^0$
64,2	77,1	part. + phot. Prod.	$158,7 = \underset{\text{part.}}{145,8} + \underset{\text{phot.}}{12,9} \hat{=} \pi^0 + \gamma$
78,4	83,3	phot. prod.	$138,3 = 133,4 + 4,9 \hat{=} \gamma$ ⇒ π^0 has m: 134,98 ⇒ no π^0
38,5	64,6	part. + phot. prod.	$196,9 = 170,8 + 25,9 \hat{=} \pi^0 + \gamma$
60,7	84,7	phot. prod.	$154,6 = 130,6 + 14 \hat{=} \gamma$
75,5	85,7	phot. prod.	$138,8 = 128,6 + 10,2 \hat{=} \gamma$
29,7	80,5	part. + phot. prod.	$189,8 = 139 + 50,8 \hat{=} \pi^0 + \gamma$
59,7	88,1	phot. prod.	152,2 = 123,8 + 2[illegible],4 $\hat{=} \gamma$
73,7	87,3	phot. prod.	$139 = 125,4 + 4,6 \hat{=} \gamma$
25,9	90,1	phot. prod.	$184 = 119,8 + 64,2 \hat{=} \gamma$
23,7	96,4	phot. prod.	$179,9 = 107,2 + 72,7 \hat{=} \gamma$