# Hadron and Nuclear Physics

# Exercise sheet 2

Winter term 2013/14

Hand-in: Monday, Nov 18th 2013, Room 108

## Problem 2.1: Magnetic Dipole Moments of Nucleons (9 points)

(a) Calculate the magnetic dipole moments of protons and neutrons considering the following information:

- The three constituent quarks carry one third of the entire nucleon mass.
- Quarks are pointlike fermions whose magnetic dipole moment can be calculated in analogy to the electron from their spin, charge and mass (Dirac particles).
- The magnetic moment of a three-quark state is obtained from the following quantum-mechanical relation:

$$\mu = 2\langle q_1 q_2 q_3 | \mu_1 S_{z1} + \mu_2 S_{z2} + \mu_3 S_{z3} | q_1 q_2 q_3 \rangle = 2 \sum_{i=1}^{3} \langle q_i | \mu_i S_z | q_i \rangle$$

- The wave function of a nucleon with spin $\uparrow$ is given by:

$$\begin{aligned} |N_\uparrow\rangle = & \sqrt{2/9}\,\left(|1_\uparrow 2_\uparrow 3_\downarrow\rangle + |3_\downarrow 1_\uparrow 2_\uparrow\rangle + |2_\uparrow 3_\downarrow 1_\uparrow\rangle\right) \\ & - \sqrt{1/18}\,\left(|1_\downarrow 2_\uparrow 3_\uparrow\rangle + |3_\uparrow 1_\downarrow 2_\uparrow\rangle + |2_\uparrow 3_\uparrow 1_\downarrow\rangle + |1_\uparrow 2_\downarrow 3_\uparrow\rangle + |3_\uparrow 1_\uparrow 2_\downarrow\rangle + |2_\downarrow 3_\uparrow 1_\uparrow\rangle\right) \end{aligned}$$

  (where for the proton $|123\rangle \equiv |uud\rangle$, for the neutron $|123\rangle \equiv |ddu\rangle$)
  Compare the magnetic moments and their ratio with the experimental values for neutron and proton.

## Problem 2.2: Exotic States (9 points)

(a) Which of these objects can exist? Why?

$$\bar{b}c, \quad uug, \quad \bar{c}cg, \quad gg, \quad ggg, \quad \bar{u}\bar{u}dd, \quad uudd, \quad uudd\bar{s}, \quad uuddss$$

(b) In which kind of reactions do you expect to find gluons and hybrids? How can you distinguish between a hybrid and a meson? Why is this so difficult?

## Problem 2.3: Missing Mass (9 points)

(a) In an experiment protons with a kinetic energy of 300 MeV are fired at a hydrogen target at rest. The outgoing protons are detected in two detectors, which are arranged symmetrically around the beam at an angle of 12°. Pairs of protons with the energies, which you can find in the table, are measured. Which reaction do occur, elastic scattering, photon production or production of a particle (if so, which particle?)?

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T_3$/MeV | 80.9 | 70 | 50.2 | 64.2 | 78.4 | 38.5 | 60.7 | 75.5 | 29.7 | 59.7 | 73.7 | 25.9 | 23.7 |
| $T_4$/MeV | 80.9 | 70 | 50.2 | 77.1 | 83.3 | 64.6 | 84.5 | 85.7 | 80.5 | 88.1 | 87.3 | 90.1 | 96.4 |

## Problem 2.4: Charmonium (9 points)

(a) Draw the level scheme of charmonium. Which quantum numbers do the different states have?

(b) Compared to the energy levels of positronium, the scheme of charmonium is very similar for the low lying states ($n = 1, 2$), whereas for higher lying states the behaviour of positronium and charmonium is very different. What can one conclude from this observation?

(c) Why are the partial widths of the $\phi$ ($\Gamma(\phi \to 3\pi) \approx 650\,\text{keV}$) and the $J/\psi$ ($\Gamma(J/\psi \to \text{hadrons}) \approx 82\,\text{keV}$) so small compared to the width of the $\rho$ ($\Gamma(\rho \to \pi\pi) \approx 150\,\text{MeV}$)? Draw the decays schematically with quark and gluon lines.

(d) In the BELLE-Collaboration, the reaction $e^+e^- \to J/\psi X$ was studied. Here, the $J/\psi$ was selected with a momentum of $|\vec{p}_{\text{CMS}}| > 2\,\text{GeV}$.

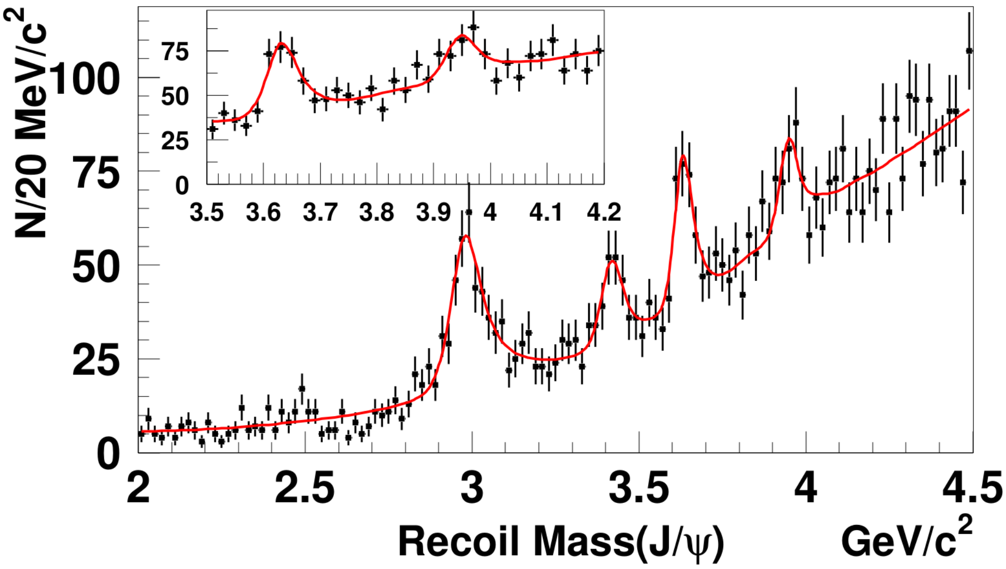



Figure 1: Mass distribution $M_{\text{Recoil}}(J/\psi) = \sqrt{(E_{\text{CMS}} - E_{J/\psi})^2 - \vec{p}^{\,2}_{J/\psi}}$ in the reaction $e^+e^- \to J/\psi X$ (K. Abe et al., BELLE-Collaboration, arXiv:hep-ex/0507019).

- Why did one use this momentum cut?
- Which quark content does the $X$ have?
- Interpret the mass distribution in the picture. Which particles can you see there? Which spin, relative orbital and total angular momentum do they have?

# 2 Übung: Besprechung

## 2.1 Aufgabe

$\Rightarrow$ Griffith

$$|p>= \underbrace{\frac{2}{3\sqrt{2}}|u_\uparrow u_\uparrow d_\downarrow>}_{a} - \underbrace{\frac{1}{3\sqrt{2}}|u_\uparrow u_\downarrow d_\uparrow>}_{b} - \underbrace{\frac{1}{3\sqrt{2}}|u_\downarrow u_\uparrow d_\uparrow>}_{c} + \underbrace{permutation}_{\Rightarrow 3*(a+b+c)}$$

$$\hat{S}_z|q_\uparrow>=\hat{S}_z|q_i \underbrace{\frac{1}{2}}_{S}, \underbrace{\frac{1}{2}}_{m_s}>=\frac{\hbar}{2}|q_p>$$

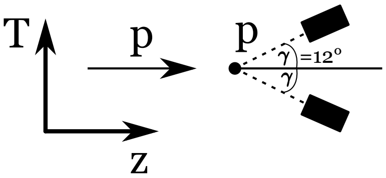
$\mu_A = (\frac{2}{3\sqrt{2}})^2 \cdot 2\dot{\sum_i} < u_\uparrow u_\uparrow d_\downarrow|\mu_i \hat{S}_z|u_\uparrow u_\uparrow d_\downarrow >= \frac{2}{9}\cdot 2(\frac{\mu_u}{2}+\frac{\mu_u}{2}-\frac{\mu_d}{2})$
$\mu_B = (\frac{1}{3\sqrt{2}})^2 \cdot 2\dot{\sum_i} < u_\uparrow u_\downarrow d_\uparrow|\mu_i \hat{S}_z|u_\uparrow u_\downarrow d_\uparrow >= \frac{1}{18}\cdot 2(\frac{\mu_u}{2}-\frac{\mu_u}{2}+\frac{\mu_d}{2}) = \frac{1}{18}\mu_d$
$\Rightarrow \mu_C = \mu_B = \frac{1}{18}\mu_d$
$\mu_p = 3(\frac{2}{q}(2\mu_u - \mu_d) + \frac{1}{18}\mu_d + \frac{1}{18}\mu_d) = \frac{1}{3}(4\mu_u - \mu_d) \quad \Rightarrow \mu_d = \frac{1}{3}(4\mu_d - \mu_u)$
Vereinfachungen:$m_u = m_d = m_q = \frac{1}{3}m_N$ , Dirac-Teilchen: $\mu = \frac{Q\hbar}{2mc}$
Kernmagneton (Wenn Photon Dirac-Teilchen):$\mu_N = \frac{e\hbar}{2m_p c}$
$\Rightarrow \mu_u = \frac{2}{3}\frac{e\hbar}{\frac{1}{3}m_p c} = 2\mu_N$ , $\mu_d = -\frac{1}{3}\frac{e\hbar}{\frac{1}{3}m_p c} = -\mu_N$
$\mu_p = \frac{1}{3}(4\mu_u - \mu_d) = 3\mu_N$ $(2.793\mu_N)$ , $\mu_n = \frac{1}{3}(4\mu_d - \mu_u) = -2\mu_N$ $(-1.913\mu_N)$
$\frac{\mu_u}{\mu_d} = -\frac{2}{3}$ (0.68)

## 2.2 Aufgabe

a) Farbladung! (e.g. uug verboten, da u je 1 Farbe, g jedoch Farbe + Antifarbe)

b) Was ist uuddss? $p\Xi^-, n\Xi^0, \Lambda\Lambda, \Sigma^+\Sigma^-$ (DiBaryon)
Beispiel: $\pi^0 \to \gamma\gamma$ : $J^{PC}: 0^{-+} \to (1^{--})^2$ (nicht $\$gamma^3$!)
$\varrho \to \pi^+\pi^-$ : $P(\varrho^0) = \eta_{\pi^+}\eta_{\pi^-}\cdot(-1)^l$
$P(\varrho) = -1 \Rightarrow l = 1, 3, \ldots$ , $P(\varrho) = 1 \Rightarrow l = 2, 4, \ldots$
Glueball:
zerfällt nur stark $\Rightarrow$ schnell $\Rightarrow$ Breite $\Rightarrow$ Andere Kanäle überlagern
Aber: Flavorblind!
Bsp: $m(G) = 20GeV$
$Br(G \to B^0\bar{B}^0) \approx Br(G \to D^0\bar{D}^0) \approx Br(G \to K^0\bar{K}^0)$

## 2.3 Aufgabe




Rutherford: $\sigma(\vartheta) = (\frac{Q_1 Q_2}{4T})^2 \frac{1}{\sin^4(\frac{\vartheta}{2}}$
Fällt sehr schnell ab: $\frac{\sigma(12°)}{\sigma(13°)} = 5\cdot 10^{-5}$
Missing Mass! Energieerhaltung: $T + 2m_p = T_3 + T_4 + 2m_p + E_x \quad \Rightarrow E_x = T - T_3 - T_4$
Nun 4-Impuls-Erhaltung:
$p_3 = \sqrt{(T_3 + m_3)^2 - m_p^2}$, $p_4 = \sqrt{(T_4 + m_4)^2 - m_p^2}$
Transversal: $p_T = 0 = p_{T,3} + p_{T,4} + p_{T,X} \quad \Rightarrow p_{T,X} = -p_{T,3} - p_{T,4}$
Longitudinal: $p_Z = p_{Z,3} + p_{Z,4} + p_{Z,X} \quad \Rightarrow p_{Z,X} = p_Z - p_{Z,3} - p_{Z,4}$
$p_{T,3} = \sin(\vartheta)p_3$ , $p_{Z,3} = \cos(\vartheta)p_3$ , $p_X^2 = p_{T,X}^2 + p_{Z,X}^2 \quad \Rightarrow M_x = \sqrt{E_x^2 - p_x^2}$
$M_x \approx m_\pi \quad \Rightarrow \quad pp \to pp\pi^0$ , $M_x \approx 0 \quad \Rightarrow \quad pp \to pp\gamma$

## 2.4 Aufgabe

$e^+e^- \to Y(4s) \to J/\psi\ X$
$X = \pi^+\pi^-$ , $\pi^0\pi^0$ , $\eta_c$ , $\eta_c\pi^+\pi^-, \ldots (J/\psi \to \mu^+\mu-$ : Br$\approx 10^{-4})$
CMS: $\begin{pmatrix} E_{CMS} \\ \vec{0} \end{pmatrix} = \begin{pmatrix} E_\psi \\ \vec{p}_\psi \end{pmatrix} + \begin{pmatrix} E_x \\ \vec{p}_x \end{pmatrix} \quad \Rightarrow \quad (E_{CMS} - E_\psi)^2 - \vec{p}_\psi^2 = M_x^2$