Hadron and Nuclear Physics

Exercise sheet 3

Winter term 2013/14

Hand-in: Monday, Dec 2nd 2013, **13:00**, Room 108

Problem 3.1: Cross-section of Electron Scattering (12 points)

An electron with a kinetic energy of 400 MeV is scattered on a ^{4}He nucleus. Calculate the differential cross section for elastic scattering at $\vartheta_{\text{Lab}} = 4°,\ 30°,\ 90°$ for the following assumptions:

(a) Both, the electron and the and the ^{4}He nucleus are point-like spinless particles.
(b) In addition to a, consider the spin of the electron.
(c) In addition to a and b, consider the finite size of the ^{4}He nucleus using a gaussian shape for the charge distribution with a half-density radius $R_{\frac{1}{2}} = 1.3\,\text{fm}$.

Problem 3.2: Form Factors (12 points)

The differential cross section $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$ for elastic scattering of electrons ($s = 1/2$) on extended nuclei with isospin $I = 0$ is described by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Mott}} \cdot |F(\boldsymbol{q})|^2\,, \qquad \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Mott}} = \frac{4Z^2\alpha^2\,(\hbar c)^2\,E'^2}{|\boldsymbol{q}c|^4}\cos\left(\frac{\vartheta}{2}\right),$$

where $|\boldsymbol{q}| = 2\,|\boldsymbol{p}'|\sin(\frac{\vartheta}{2})$ is the three-momentum transfer, E' the energy and $\boldsymbol{p}'$ the momentum of the outgoing electron. The relation between the form factor $F(\boldsymbol{q})$ and charge distribution $\rho(\boldsymbol{r})$ is given by

$$F(\boldsymbol{q}) = \int \mathrm{d}^3r\,\rho(\boldsymbol{r}) \cdot \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}\ . \tag{1}$$

(a) Show that for a charge distribution with spherical symmetry, equation 1 simplifies to

$$F(q) = 4\pi \int_0^\infty r^2\mathrm{d}r\,\rho(r)\frac{\sin(qr)}{qr}\ .$$

(b) In first order most nuclei can be described by homogenous charged spheres with radius R

$$\rho(r) = \begin{cases} \rho_0 & \text{if } r \le R \\ 0 & \text{if } r > R\ . \end{cases}$$

Use the normalization

$$\int \rho(r)\mathrm{d}^3r = 1$$

and show that the form factor is given by

$$F(q) = \frac{3}{q^3R^3}\left(\sin(qR) - qR\cos(qR)\right)\ .$$

Problem 3.3: J/ψ Decays (12 points)

(a) The J/ψ decays for example in the meson pairs $\phi\eta$, $\omega\eta$, $\phi\eta'$ and $\omega\eta'$. Which ratio of the decay rates $\frac{\Gamma(J/\psi\to\phi\eta)}{\Gamma(J/\psi\to\omega\eta)}$ and $\frac{\Gamma(J/\psi\to\phi\eta')}{\Gamma(J/\psi\to\omega\eta')}$ would you expect? Use the following wavefunctions:

$$|\,\eta\rangle = \cos(\theta)\,|\,\eta_8\rangle - \sin(\theta)\,|\,\eta_1\rangle \qquad |\,\omega\rangle = \frac{1}{\sqrt{2}}\left(|\,u\bar{u}\rangle + |\,d\bar{d}\rangle\right)$$

$$|\,\eta'\rangle = \sin(\theta)\,|\,\eta_8\rangle + \cos(\theta)\,|\,\eta_1\rangle \qquad |\,\phi\rangle = |\,s\bar{s}\rangle\ .$$

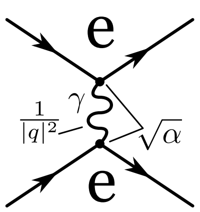
The pseudoscalar mixing angle is $\theta = -23°$ and the octet and singlet-wavefunctions are given by

$$\begin{aligned} \mid \eta_1\rangle &= \frac{1}{\sqrt{3}}\left(\mid u\bar{u}\rangle + \mid d\bar{d}\rangle + \mid s\bar{s}\rangle\right) \\ \mid \eta_8\rangle &= \frac{1}{\sqrt{6}}\left(\mid u\bar{u}\rangle + \mid d\bar{d}\rangle - 2 \mid s\bar{s}\rangle\right) \ . \end{aligned}$$

Compare your result for the reaction rate ratios with the corresponding PDG values.

3 Übung: Besprechung

3.1 Aufgabe



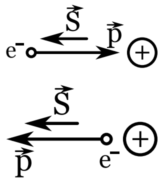
a) $\frac{d\sigma}{d\Omega}_{Rutherford} = \frac{4Z^2\alpha^2(\hbar c)E'^2}{|\vec{q}c|^4} \overset{\hbar=c=1}{=} \frac{\alpha^2}{T^2\sin^4(\frac{\vartheta}{2})}$

Dabei gilt: $E >> m \quad \Rightarrow \quad E \approx T \approx p$ und $1 = \hbar c = 197\text{MeV}\cdot\text{fm}$

$\Rightarrow$

ϑ	4°	30°	90°
$\frac{d\sigma}{d\Omega}$	87.1 mb	28.8 μb	0.517 μb

b)



Helizität $h = \frac{\vec{s}\vec{p}}{|\vec{s}||\vec{p}|} = -1$

$h = 1$

$\frac{d\sigma}{d\Omega}_{Mott} = \frac{d\sigma}{d\Omega}_{Ruth} \cdot (1 - \beta^2 \sin^2(\frac{\vartheta}{2})) = \frac{d\sigma}{d\Omega}_{Ruth} \cdot (\cos^2(\frac{\vartheta}{2}))$

$\Rightarrow$

ϑ	4°	30°	90°
$\frac{d\sigma}{d\Omega}$	87.01 mb	26.87 μb	0.258 μb

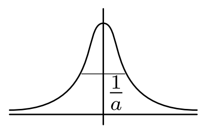
c) $F(q) = \int d^3r\varrho(r)e^{i\vec{q}\vec{r}}$ mit $\int_0^\infty f(r)dr = 1$.

$f(r) = (\frac{a^2}{2\pi})^{\frac{2}{3}} \exp(-\frac{1}{2}a^2r^2)$

$\frac{f(0.13\text{ fm})}{f(0\text{ fm})} \overset{!}{=} \frac{1}{2} = \frac{\exp(-\frac{1}{2}a^2\cdot(1.3\text{ fm})^2)}{\exp(0)} \quad \Rightarrow a = 0.906\text{ fm} \approx 178.5\text{ MeV}$

$\Rightarrow F(q) = \exp(-\frac{1}{2}\frac{q^2}{a^2}) = \exp(-10.04 \cdot \sin^2(\frac{\vartheta}{2}))$

$\frac{d\sigma}{d\Omega}_{exp} = \frac{d\sigma}{d\Omega}_{Mott} \cdot |F(q)|^2$



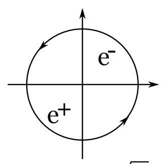
$\Rightarrow$

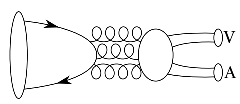
ϑ	4°	30°	90°
$\frac{d\sigma}{d\Omega}$	84.9 mb	7.0 μb	11.2 μb
$F(q)$	0.988	0.510	0.0.0066

3.2 Aufgabe

siehe Abgabe

3.3 Aufgabe



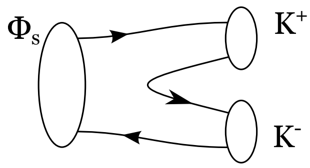


$\Gamma \propto |\Psi(0)|^2$

$|\eta >= A|n_8 > +B|n_1 >$, $\quad A^2 + B^2 \overset{!}{=} 1$

$< \phi|\eta_1 >= \frac{1}{\sqrt{3}}$, $< \omega|\eta_1 >= \sqrt{\frac{2}{3}}$, $< \phi|\eta_8 >= -\sqrt{\frac{2}{3}}$, $< \omega|\eta_8 >= \frac{1}{\sqrt{3}}$

Rechnerisch :	*PDG :*
$\frac{\|<\phi\|\eta>\|^2}{\|<\omega\|\eta>\|^2} = 0.38$	$\frac{7.5\cdot10^{-4}}{1.74\cdot10^{-3}} = 0.43$
$\frac{\|<\phi\|\eta'>\|^2}{\|<\omega\|\eta'>\|^2} = 2.61$	$\frac{4\cdot10^{-4}}{1.82\cdot10^{-3}} = 2.20$



$\beta_\nu = 50\%$

$< K^T|K^T >\approx 0$