Higher hadron physics

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Problem 4.7

$$\sigma(E) = \frac{2J+1}{16\pi(\sqrt{s})^2} \frac{\Gamma_i \Gamma_j}{(E-E_0)^2 + \frac{\Gamma^2}{4}} \quad , \quad \Gamma_{ee} = \Gamma_{\mu\mu}$$

$$J = 1 , \quad E_0 \approx \sqrt{s} , \quad \Gamma = \Gamma_{\mu\mu} + \Gamma_h + \Gamma_{ee} = 2\Gamma_{\mu\mu} + \Gamma_h$$

$$\int_0^\infty \sigma_{\mu\mu}(E)\, dE = \frac{3\Gamma_{\mu\mu}^2}{16\pi \cdot s} \cdot \int_0^\infty \frac{1}{(E-\sqrt{s})^2 + \frac{(2\Gamma_{\mu\mu}+\Gamma_h)^2}{4}} dE$$

$$= \frac{3\Gamma_{\mu\mu}^2}{16\pi \cdot s} \cdot \frac{2}{2\Gamma_{\mu\mu}+\Gamma_h} \cdot \left(-\arctan(-\infty) + \arctan\left(\frac{\sqrt{s}}{\frac{2\Gamma_{\mu\mu}+\Gamma_h}{2}}\right)\right)$$

$\arctan\left(\frac{\sqrt{s}}{\Gamma}\right) \approx \arctan(\infty)$, since $\sqrt{s} \gg \Gamma$:

As the total partial width is much smaller as the beam's energy (9,5 GeV), the ratio can be considered a very high number which leads arctan to be very near its asymptotic value against ∞: $\frac{\pi}{2}$.

$$\Rightarrow \int_0^\infty \sigma_{\mu\mu}(E)\, dE = \frac{3\Gamma_{\mu\mu}^2}{16\pi \cdot s} \cdot \left(\frac{\pi}{2} + \frac{\pi}{2}\right) \cdot \frac{2}{2\Gamma_{\mu\mu}+\Gamma_h} = \frac{3}{8s} \frac{\Gamma_{\mu\mu}^2}{2\Gamma_{\mu\mu}+\Gamma_h}$$

analog: $$\int_0^\infty \sigma_h(E)\, dE = \frac{3}{8s} \frac{\Gamma_{\mu\mu}\Gamma_h}{2\Gamma_{\mu\mu}+\Gamma_h}$$

$$\frac{\int_0^\infty \sigma_{\mu\mu}(E)\, dE}{\int_0^\infty \sigma_h(E)\, dE} = \frac{\Gamma_{\mu\mu}}{\Gamma_h} \overset{!}{=} \frac{8{,}5 \cdot 10^{-33}\, cm^2 MeV}{3{,}3 \cdot 10^{-31}\, cm^2 MeV} = 0{,}025758$$

$$\Rightarrow \frac{3}{8s} \frac{\Gamma_{\mu\mu}^2}{2\Gamma_{\mu\mu} + \Gamma_{\mu\mu}/0{,}025758} \overset{!}{=} 8{,}5 \cdot 10^{-23}\, cm^2 MeV$$

$(9500\ MeV)^2$

$$\Leftrightarrow \quad \Gamma_{\mu\mu} = \frac{8 \cdot (9500\ MeV)^2}{3}\left(2 + \frac{1}{0{,}025758}\right) \cdot 8{,}5 \cdot 10^{-33}\, cm^2 MeV$$

$$= 8{,}35113 \cdot 10^{-23}\, MeV^3 cm^2$$

$$\Rightarrow \quad \Gamma_h = (0{,}025758)^{-1}\, \Gamma_{\mu\mu} = 3{,}2422 \cdot 10^{-21}\, cm^2 MeV$$

natural units: $\frac{1}{1\,MeV} \mathrel{\hat{=}} 1{,}973 \cdot 10^{-11}\, cm$, $\frac{1}{keV} \mathrel{\hat{=}} 6{,}5821 \cdot 10^{-19}\, s$

$$\Rightarrow \quad \Gamma_{\mu\mu} \mathrel{\hat{=}} 214{,}473\ keV \mathrel{\hat{=}} 0{,}141169\ a\ Hz$$

$$\Gamma_h \mathrel{\hat{=}} 8326{,}59\ keV \mathrel{\hat{=}} 5{,}48066\ a\ Hz$$

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Problem 4.3

$$\frac{d^2\sigma}{d\Omega dE'} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\left(1+2\tau\tan^2\left(\frac{\vartheta}{2}\right)\right)$$

$$\frac{d^2\sigma}{d\Omega dE'} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\left(W_2+2W_1\tan^2\left(\frac{\vartheta}{2}\right)\right)$$

compact notation : $W_1 \hat{=} W_1(Q^2,\nu)$

compare equation's coefficients :

$$\Rightarrow \quad 1 \overset{!}{=} W_2 \quad , \quad \tau \overset{!}{=} W_1$$

$$F_1 = M W_1 = M\tau$$

$$F_2 = \nu W_2 = \nu$$

$$\nu = \frac{Q^2}{2Mx} \quad , \quad \tau = \frac{Q^2}{4m^2} = \frac{Q^2}{4M^2x^2}$$

As in DIS interaction occurs directly with quarks, the total Nucleon's mass has to be scaled with the momentum's fraction x : $m = Mx$

$$\Rightarrow \quad \frac{F_1}{F_2} = \frac{M\tau}{\nu} = \frac{\frac{Q^2}{4Mx^2}}{\frac{Q^2}{Mx}} = \frac{1}{2x}$$

$$\Rightarrow \quad 2x F_1 = F_2$$

$\Rightarrow$ Callan - Gross