

Problem 4.7

$$\sigma(E) = \frac{2J+1}{16\pi(\sqrt{s})^2} \frac{\Gamma_i \Gamma_j}{(E-E_0)^2 + \frac{\Gamma^2}{4}}, \quad \Gamma_{ee} = \Gamma_{\mu\mu}$$

$$J = 1, \quad E_0 \approx \sqrt{s}, \quad \Gamma = \Gamma_{\mu\mu} + \Gamma_h + \Gamma_{ee} = 2\Gamma_{\mu\mu} + \Gamma_h$$

$$\int_0^{\infty} \sigma_{\mu\mu}(E) dE = \frac{3\Gamma_{\mu\mu}^2}{16\pi \cdot s} \cdot \int_0^{\infty} \frac{1}{(E-\sqrt{s})^2 + \frac{(2\Gamma_{\mu\mu} + \Gamma_h)^2}{4}} dE$$

$$= \frac{3\Gamma_{\mu\mu}^2}{16\pi \cdot s} \cdot \frac{2}{2\Gamma_{\mu\mu} + \Gamma_h} \cdot \left(-\arctan(-\infty) + \arctan\left(\frac{\sqrt{s}}{\frac{2\Gamma_{\mu\mu} + \Gamma_h}{2}}\right) \right)$$

$$\arctan\left(\frac{\sqrt{s}}{\Gamma}\right) \approx \arctan(\infty), \text{ since } \sqrt{s} \gg \Gamma:$$

As the total partial width is much smaller as the beam's energy (9,5 GeV), the ratio can be considered a very high number which leads arctan to be very near its asymptotic value against ∞ : $\frac{\pi}{2}$.

$$\Rightarrow \int_0^{\infty} \sigma_{\mu\mu}(E) dE = \frac{3\Gamma_{\mu\mu}^2}{16\pi \cdot s} \cdot \left(\frac{\pi}{2} + \frac{\pi}{2}\right) \cdot \frac{2}{2\Gamma_{\mu\mu} + \Gamma_h} = \frac{3}{8s} \frac{\Gamma_{\mu\mu}^2}{2\Gamma_{\mu\mu} + \Gamma_h}$$

$$\text{analog: } \int_0^{\infty} \sigma_h(E) dE = \frac{3}{8s} \frac{\Gamma_{\mu\mu} \Gamma_h}{2\Gamma_{\mu\mu} + \Gamma_h}$$

$$\frac{\int_0^{\infty} \sigma_{\mu\mu}(E) dE}{\int_0^{\infty} \sigma_h(E) dE} = \frac{\Gamma_{\mu\mu}}{\Gamma_h} \stackrel{!}{=} \frac{8,5 \cdot 10^{-33} \text{ cm}^2 \text{ MeV}}{3,3 \cdot 10^{-31} \text{ cm}^2 \text{ MeV}} = 0,025758$$

$$\Rightarrow \frac{3}{8s} \frac{\Gamma_{\mu\mu}^2}{2\Gamma_{\mu\mu} + \Gamma_h} \stackrel{!}{=} 8,5 \cdot 10^{-23} \text{ cm}^2 \text{ MeV}$$

$$\begin{aligned} \Rightarrow \Gamma_{\mu\mu} &= \frac{8 \cdot (9500 \text{ MeV})^2}{3} \left(2 + \frac{1}{0,025758}\right) \cdot 8,5 \cdot 10^{-33} \text{ cm}^2 \text{ MeV} \\ &= 8,35113 \cdot 10^{-23} \text{ MeV}^3 \text{ cm}^2 \end{aligned}$$

$$\Rightarrow \Gamma_h = (0,025758)^{-1} \Gamma_{\mu\mu} = 3,2422 \cdot 10^{-21} \text{ cm}^2 \text{ MeV}$$

$$\text{natural units: } \frac{1}{\text{MeV}} \hat{=} 1,973 \cdot 10^{-11} \text{ cm}, \quad \frac{1}{\text{keV}} \hat{=} 6,5821 \cdot 10^{-19} \text{ s}$$

$$\Rightarrow \Gamma_{\mu\mu} \hat{=} 214,473 \text{ keV} \hat{=} 0,141169 \text{ a Hz}$$

$$\Gamma_h \hat{=} 8326,59 \text{ keV} \hat{=} 5,48066 \text{ a Hz}$$

Problem 4.3

$$\frac{d^2\sigma}{d\Omega dE'} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{mott}} (1 + 2\tau \tan^2(\frac{\nu}{2}))$$

$$\frac{d^2\sigma}{d\Omega dE'} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{mott}} (W_2 + 2W_1 \tan^2(\frac{\nu}{2}))$$

compact notation: $W_1 \hat{=} W_1(Q^2, \nu)$

compare equation's coefficients:

$$\Rightarrow 1 \hat{=} W_2, \quad \tau \hat{=} W_1$$

$$F_1 = M W_1 = M \tau$$

$$F_2 = \nu W_2 = \nu$$

$$\nu = \frac{Q^2}{2Mx}, \quad \tau = \frac{Q^2}{4m^2} = \frac{Q^2}{4M^2x^2}$$

As in DIS interaction occurs directly with quarks, the total Nucleon's mass has to be scaled with the momentum's fraction x : $m = Mx$

$$\Rightarrow \frac{F_1}{F_2} = \frac{M\tau}{\nu} = \frac{\frac{Q^2}{4M^2x^2}}{\frac{Q^2}{2Mx}} = \frac{1}{2x}$$

$$\Rightarrow 2x F_1 = F_2 \quad \Rightarrow \text{Callan-Gross}$$