Hadron and Nuclear Physics

Exercise sheet 4

Winter term 2013/14

Hand-in: Monday, Dec 16th 2013, **13:00**, Room 108

Problem 4.1: Breit-Wigner Distribution (9 points)

In e^+e^- annihilation, a small resonance (smaller than the intrinsic energy spread of the beam) was observed at $\sqrt{s} = 9.5\,\mathrm{GeV}$ in the reactions

$$e^+e^- \to \mu^+\mu^- \qquad \text{and} \qquad e^+e^- \to \text{hadrons}.$$

The integrated cross section was measured to

$$\int \sigma_{\mu\mu}(E)\mathrm{d}E = 8.5 \cdot 10^{-33}\,\mathrm{cm^2 MeV}$$

and

$$\int \sigma_{\mathrm{h}}(E)\mathrm{d}E = 3.3 \cdot 10^{-31}\,\mathrm{cm^2 MeV}.$$

(a) Calculate with the Breit-Wigner-Formula

$$\sigma(E) = \frac{2J+1}{16\pi\left(\sqrt{s}\right)^2}\frac{\Gamma_i\Gamma_f}{(E-E_0)^2+\frac{\Gamma^2}{4}}$$

the partial widths $\Gamma_{\mu\mu}$ and Γ_{h} ($J=1$).
Hint: $\Gamma_{\mathrm{ee}} = \Gamma_{\mu\mu} = \Gamma_{\tau\tau}$ and $\int \frac{\mathrm{d}x}{(x-a)^2+b^2} = \frac{\arctan\frac{a-x}{b}}{b}$

Problem 4.2: Inelastic Neutrino Scattering in the Quark Model (9 points)

Consider the scattering of muon-neutrinos on free, massless quarks. We will simplify things and discuss only strangeness-conserving reactions, i.e. transitions only between the u and the d quarks.

(a) Write down all the possible charged- current elastic reactions for both ν_μ and $\bar{\nu}_\mu$ on the u and d quarks as well as the $\bar{\mathrm{u}}$ and $\bar{\mathrm{d}}$ antiquarks. (There are four such reactions).

(b) Give helicity arguments to predict the angular distribution for each of the reactions.

(c) Assume that inelastic ν (or $\bar{\nu}$)-nucleon cross sections are given by the sum of the cross sections for the four processes that have been listed above. Derive the quark model predictions for the ratio of the total cross section of antineutrino-nucleon scattering compared with neutrino-nucleon scattering, $\sigma^{\bar{\nu}\mathrm{N}}/\sigma^{\nu\mathrm{N}}$.
Hint: The differential cross section for (anti)-neutrino d-quark scattering is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{G_{\mathrm{F}}^2 S}{4\pi^2}\cos^2\theta_{\mathrm{C}}$$

the one for (anti)-neutrino u-quark scattering by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{G_{\mathrm{F}}^2 S}{16\pi^2}\cos^2\theta_{\mathrm{C}}\,(1-\cos\vartheta)^2$$

(d) The experimental value is $\sigma^{\bar{\nu}\mathrm{N}}/\sigma^{\nu\mathrm{N}} = 0.37 \pm 0.02$. What does this value tell you about the quark/antiquark structure of the nucleon?

Problem 4.3: Callan-Gross-Relation (9 points)

For the deep inelastic scattering off the proton, one gets for the structure functions of the protons the relation

$$2xF_1(x,Q^2) = F_2(x,Q^2)$$

which is known as Callan-Gross-Relation. $F_2(x, Q^2)$ and $F_1(x, Q^2)$ describe the electric and magnetic part of the interaction. For the scattering of two pointlike spin 1/2 particles one obtains

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega \mathrm{d}E'} = \left(\frac{\mathrm{d}}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} \left[1 + 2\tau \tan^2\frac{\vartheta}{2}\right]$$

The factor $\tau = \frac{Q^2}{4M^2c^2}$ gives the part of the magnetic moment of the target with mass M to the interaction.

(a) Derive from the comparison of this scattering cross section with the cross section for deep inelastic scattering

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega \mathrm{d}E'} = \left(\frac{\mathrm{d}}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} \left[W_2(Q^2, \nu) + 2 \cdot W_1(Q^2, \nu) \tan^2\frac{\vartheta}{2}\right]$$

the Calan-Gross-Relation.

Problem 4.4: Deep Inelastic Scattering (9 points)

(a) Use the deep inelastic lepton-nucleon-scattering in a reference system in which the nucleon has a high momentum. Show that for an infinite momentum the Bjørken-Variable $x = \frac{Q^2}{2M\nu}$ is identical to the longitudinal momentum fraction ξ of the hit parton.
Hint: Use a relation of x and ξ, which contains the leading order of the dependency of the nucleon mass M and the parton mass m. Ignore transverse momenta.

4 Übung: Besprechung

4.1 Aufgabe

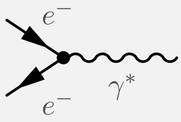
Hinweis

Zusammenhang mit Spins:

$\sigma(E)\frac{J+1}{4\pi s(2S_A+1)(2S_B+1)}\frac{\Gamma^2}{(E-E_0)^2+\frac{\Gamma^2}{4}}$

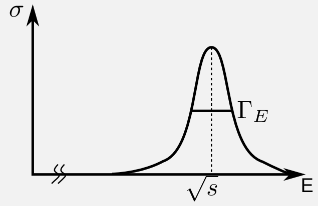
$\overset{A}{\rightarrow}\overset{B}{\leftarrow}$. Naiv: $\uparrow\uparrow$, $\uparrow\downarrow$, $\downarrow\uparrow$, $\downarrow\downarrow$

QM:Triplett ($J = 1$), Singulett ($J = 0$):



$$\begin{pmatrix} E_A \\ \vec{p} \end{pmatrix} + \begin{pmatrix} E_B \\ -\vec{p} \end{pmatrix} = \begin{pmatrix} E_Y \\ \vec{0} \end{pmatrix}$$

Nährungen:



Da $\sqrt{s} \gg \Gamma_E \;\Rightarrow\; \arctan(\frac{\sqrt{s}}{\Gamma_E}) \approx \frac{\pi}{2}$

$\sigma(E)$ hat Breit–Wigner Form:

$\Rightarrow \int\limits_{\sqrt{s}-\Gamma_E}^{\sqrt{s}+\Gamma_E} \sigma(E)dE \;\rightarrow\; \int\limits_0^\infty \sigma(E)dE$

Außerdem $\Gamma_E \ll E_0 \;\Rightarrow\; \sqrt{s}$ const.

und $E_0 \approx \sqrt{s}$

Wir kürzen ab: $\sigma_{\mu\mu} =: \sigma_\mu$, $\Gamma_{ee} = \Gamma_{\tau\tau} = \Gamma_{\mu\mu} =: \Gamma_\mu$, $\int\limits_0^\infty \sigma_\mu(E)de =: \Sigma_\mu$

$\Sigma_\mu = \frac{3\Gamma_\mu^2}{16\pi s}\left[2\arctan(\frac{2(E-E_0)}{\Gamma})\right]_0^\infty \approx \frac{3\Gamma_\mu^2}{16\pi s}\frac{2\pi}{\Gamma} = \frac{3\Gamma_\mu^2}{8s\Gamma} \qquad \wedge \qquad \Sigma_h = \frac{3\Gamma_\mu\Gamma_h}{8s\Gamma}$

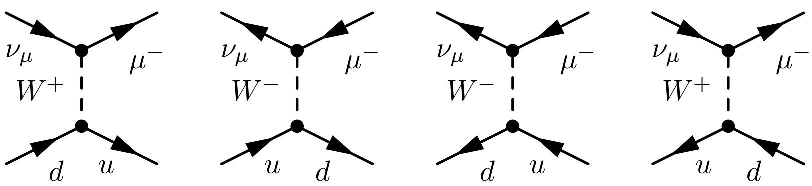
$\Rightarrow \frac{\Sigma_\mu}{\Sigma_h} = \frac{\Gamma_\mu}{\Gamma_h} \;\Rightarrow\; \Gamma_h = 38.82\Gamma_\mu \qquad \wedge \qquad \Gamma = 3\Gamma_\mu + \Gamma_h = 41.82\Gamma_\mu$

$\Sigma_\mu = \frac{\Gamma_\mu^2}{8s\cdot 41.82\Gamma_\mu} \;\Rightarrow\; \Gamma_\mu = \frac{8s\cdot 41.82\Sigma_\mu}{3} = 8.55\cdot 10^{-23}\ MeV^3\ cm^2$

$1\ cm \hat{=} \frac{1}{197\cdot 10^{-13}}\ MeV \;\Rightarrow\; \Gamma_\mu = 0.22\ MeV \qquad \wedge \qquad \Gamma_h = 8.54\ MeV$

4.2 Aufgabe

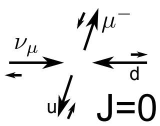
a) $\nu_\mu\ d \rightarrow \mu^-\ u$ $\quad \bar{\nu}_\mu\ u \rightarrow \mu^+\ d$ $\quad \bar{\nu}_\mu\ \bar{d} \rightarrow \mu^+\ \bar{u}$ $\quad \nu_\mu\ \bar{u} \rightarrow \mu^-\ \bar{d}$



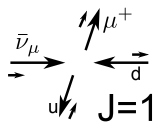
b) CMS: neutrinos immer links-, $\bar{\nu}$ immer rechtshändig

e^-: je schneller desto linkshändiger

$\nu_\mu\ d \rightarrow \mu^-\ u$:



$\bar{\nu}_\mu\ d \rightarrow \mu^+\ u$:



c) $\sigma(\nu_\mu d) = \sigma(\bar{\nu}_\mu \bar{d}) =: \sigma_d$

$\sigma_d = \int \frac{d\sigma}{d\Omega} d\cos(\vartheta)d\varphi = \frac{G_f^2 s}{\pi}\cos^2(\theta_c)$

$\sigma(\nu_\mu \bar{u}) = \sigma(\bar{\nu}_\mu u) =: \sigma_u$

$\sigma_u = \int \frac{d\sigma}{d\Omega} d\cos(\vartheta)d\varphi = \frac{G_f^2 s}{8\pi}\left[-\frac{1}{3}(1-\cos^2(\theta_c))\right]_{-1}^1 = \frac{G_f^2 s}{3\pi}\cos^2(\theta_c)$

Neutron: $\frac{\sigma(\nu_\mu\ n)}{\sigma(\bar{\nu}_\mu n)} = \frac{2\sigma(\nu_\mu\ d)}{\sigma(\bar{\nu}_\mu\ u)} = \frac{2}{\frac{1}{3}} = 6$

Proton: $\frac{\sigma(\nu_\mu\ p)}{\sigma(\bar{\nu}_\mu\ p)} = \frac{\sigma(\nu_\mu d)}{\sigma(\bar{\nu}_\mu u)} = \frac{1}{2\cdot\frac{1}{3}} = \frac{3}{2}$

Nukleon: $\frac{\sigma(\nu_\mu\ N)}{\sigma(\bar{\nu}_\mu\ N)} = \frac{\frac{1}{2}(\sigma(\nu_\mu\ p)+\sigma(\nu_\mu\ n))}{\frac{1}{2}(\sigma(\bar{\nu}_\mu\ p)+\sigma(\bar{\nu}_\mu\ n))} = \frac{\sigma(\nu_\mu\ d)+2\sigma(\nu_\mu\ d)}{2\sigma(\bar{\nu}_\mu\ u)+\sigma(\bar{\nu}_\mu\ u)} = 3$

d Der errechnete Wert von $1/3$ gilt nur, falls das Nukleon nur aus 3 Valenz-Quarks entspricht. Da die Wechselwirkung auch mit Seequarks statt findet, ist der experimentelle Wert unterschiedlich.

4.3 Aufgabe: Hinweise

Hinweis

Definition:

Proton vor dem Stoß: P

Ausgetauschtes photon: p

Proton nach dem Stoß: q

Invariante Masse: W^2

Bjorken x und Elastizität:

$W^2 = p'^2 = (P+q)^2 = M^2 + 2Pq + q^2$

$Q^2 := -q^2$, $\nu := \frac{Pq}{M}$

Das Produkt von Lorenzvektoren ist wieder Lorenzinvariant!

$W^2 = M^2 + 2M\nu - Q^2$

Dabei ist $2M\nu - Q^2 = 0$ der elastische Stoß und

$2M\nu - Q^2 > 0$ der inelastische.

Daraus lässt sich $x := \frac{Q^2}{2M\nu}$

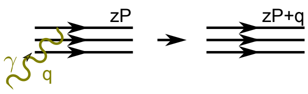
als Maß für die Elastizität einführen.

Bjorken-x: $x = 1$: elastischt $0 < x < 1$: inelastisch

Labor: $p = \begin{pmatrix} M \\ \vec{0} \end{pmatrix}$, $q = \begin{pmatrix} E - E' \\ \vec{q} \end{pmatrix}$, $\nu = \frac{Pq}{M} = E - E'$

$(\underbrace{xP}_{P_q^2})^2 = x^2M^2 \quad \Rightarrow m_q = xM$

4.4 Aufgabe



Quarks sind elementar $\Rightarrow$ inv. bleiben gleich

$m_q^2 = m_q'^2 \quad \Rightarrow \ (zP)^2 = (zP+q)^2 \quad \Rightarrow \ z^2M^2 = z^2M^2 + q^2 + 2zqP$

$\Rightarrow z = -\frac{q^2}{2qP} = \frac{Q^2}{2M\frac{qP}{M}} = \frac{Q^2}{2M\nu} = x$