Hadron and Nuclear Physics

Exercise sheet 5

Winter term 2013/14

Hand-in: Monday, Jan 13th 2014, **13:00**, Room 108

Problem 5.1: Energy- without Momentum-Transfer (9 points)

The following elementary-particle reaction may be carried out on a proton target at rest in the laboratory:

$$\mathrm{K}^- + \mathrm{p} \rightarrow \pi^0 + \Lambda$$

Find the special value of the the incident K^- energy such that the Λ can be produced at rest in the laboratory. Your answer should be expressed in terms of rest masses m_{π^0}, m_{K^-}, m_{p}, m_Λ.

Problem 5.2: Deuteron (9 points)

The deuteron is a bound state of a proton and a neutron of total angular momentum $J = 1$. It is known to be principally an S ($l = 0$) state with a small admixture of a D ($l = 2$) state.

(a) Explain why a P state cannot contribute.

(b) Explain why a G state cannot contribute.

Problem 5.3: Pion capture (9 points)

A π^- ($J^P = 0^-$) is initially bound in the lowest-energy Coulomb wave function around a deuteron. It is captured by the deuteron (a proton and a neutron in $^3S_1$ state), which is converted into a pair of neutrons:

$$\pi^- + \mathrm{d} \rightarrow \mathrm{n} + \mathrm{n}$$

(a) What is the orbital angular momentum of the neutron pair?

(b) What is the total spin S?

(c) What is the probability for finding both neutron spins directed opposite to the spin of the deuteron?

Problem 5.4: Dalitz-Plots (9 points)

CLEO-c was an experiment at the Cornell Electron Storage Ring (CESR) which examined open-charm mesons produced in the reactions $\mathrm{e}^+\mathrm{e}^- \rightarrow \psi(3770) \rightarrow \mathrm{D}^0\bar{\mathrm{D}}^0$ and $\mathrm{e}^+\mathrm{e}^- \rightarrow \psi(3770) \rightarrow \mathrm{D}^+\mathrm{D}^-$. What resonances are visible in the CLEO Dalitz-plot (and the corresponding mass projections) of the decay $\mathrm{D}^0 \rightarrow \mathrm{K}^-\pi^+\pi^0$? Also calculate the minimum and maximum allowed value of $M^2(\mathrm{K}^-\pi^+)$ and $M^2(\pi^+\pi^0)$.

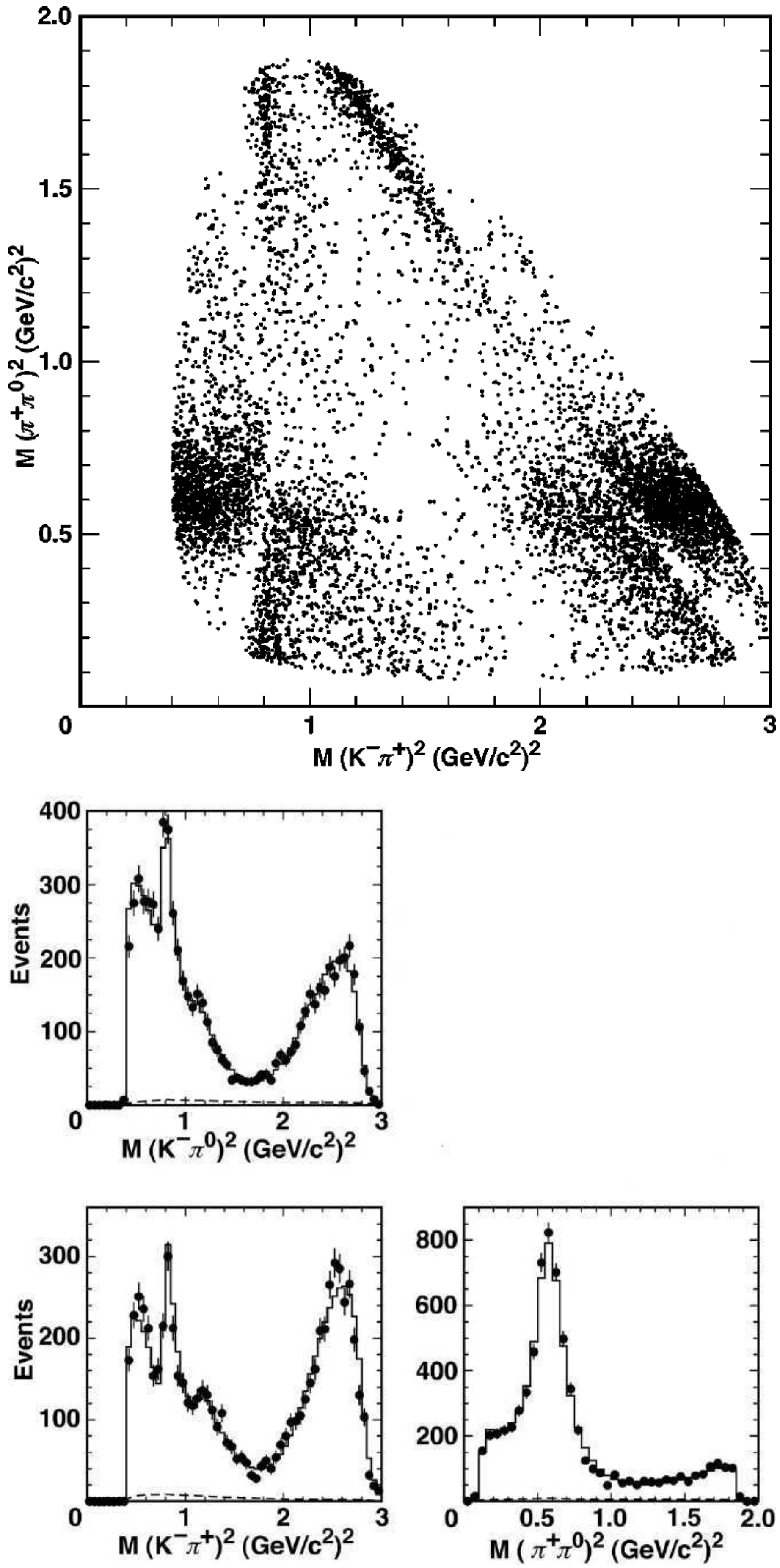
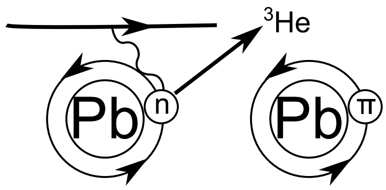


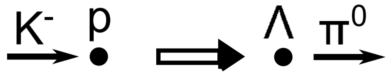
Figure 1: Dalitz-plot for the decay $D^0 \to K^-\pi^+\pi^0$ and the mass projections.

5 Übung: Besprechung



5.1 Aufgabe

$K^- \;+\; p \;\rightarrow\; \pi^0 \;+\; \Lambda$



4–Impuls: $\begin{pmatrix} E_k \\ \vec{p} \end{pmatrix} + \begin{pmatrix} m_p \\ \vec{0} \end{pmatrix} = \begin{pmatrix} m_\Lambda \\ \vec{0} \end{pmatrix} + \begin{pmatrix} E_\pi \\ \vec{p} \end{pmatrix}$

$\Rightarrow p_K + p_p = p_\Lambda + p_\pi \quad \Leftrightarrow \quad p_K^2 + p_p^2 + 2p_p p_K = p_\Lambda^2 + p_\pi^2 + 2p_\Lambda p_\pi$

$\Leftrightarrow \quad m_K^2 + m_p^2 + 2E_k m_p = m_\Lambda^2 + m_\pi^2 + 2E_\pi m_\Lambda$

$E_\pi = E_K + m_p - m_\Lambda$

$\Rightarrow \quad 2E_K m_p = m_\pi^2 + m_\Lambda^2 - m_K^2 - m_p^2 + 2E_K m_\Lambda + 2m_\Lambda m_p - 2m_\Lambda^2$

$\Leftrightarrow 2E_K(m_p - m_\Lambda) = m_\pi^2 - m_K^2 - m_\Lambda^2 - m_p^2 + 2m_\Lambda m_p$

$\Leftrightarrow E_K = \frac{m_K^2 - m_\pi^2 + (m_\Lambda - m_p)^2}{2(m_\Lambda - m_p)} = 724,21\ \text{MeV}$

5.2 Aufgabe

siehe Abgabe

5.3 Aufgabe

ab) $\pi^- \;+\; d \;\rightarrow\; n \;+\; n$

$0^- \;+\; 1^+ \rightarrow\; 0^+ + 0^+$

$\eta_\pi \cdot \eta_d \cdot (-1)^L = \eta_n^2 \cdot (-1)^{L'}$

Mit η intrinsische Parität und L Bahndrehimpuls.

Das Pion ist im niedrigsten Energiezustand $\Rightarrow L = 0$

$\Rightarrow \quad -1 = (-1)^{L'} \quad \Rightarrow \quad L' = 1, 3, 5, ...$

2 identische Fermionen $\Rightarrow$ Streuwellenfunktion Ψ asymmetrisch.

$\Psi = \Psi_{ort} \cdot \Psi_{spin}$

Negative Parität $\Rightarrow \Psi_{ort}$ asymmetrisch (-)

$\Rightarrow \Psi_{spin}$ symmetrisch (+).

$S = 0 \Rightarrow$Sigulett asymmetrisch

$S = 1 \Rightarrow$Triplett symmetrisch

$\Rightarrow S = 1.$

$|L' - 1| \leq J \leq |L' + 1|$

$\Rightarrow L' = 1$

c) 1. Möglichkeit: Betrachte z-Komponente des Spins:

$J_z = 1(oben) = L_z + S_z = L_z - 1(unten)$

$\Rightarrow L_z = 2 \notin \{-1, 0, 1\}(a)$ ↯

2. Möglichkeit: Clebsch Gordan Spin Funktionen:

$|S_n, S_{n,z}\rangle\, |S_n, S_{n,z}\rangle = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle\, |\tfrac{1}{2}, -\tfrac{1}{2}\rangle = |1, -1\rangle = |S, S_z\rangle$

$|\phi\rangle = |S, S_z\rangle\, |L, L_z\rangle = |1, -1\rangle\, |1, 1\rangle = \frac{1}{\sqrt{6}}\, |2, 0\rangle - \frac{1}{\sqrt{2}}\, |1, 0\rangle + \frac{1}{\sqrt{3}}\, |0, 0\rangle$

Aufenthaltswahrscheinlichkeit des Spinzustandes in der Reaktion: $\langle 1, 1|\phi\rangle = 0$

5.4 Aufgabe

Siehe Abgabe.
Hinweis zur Vorgehensweise: Hier zunächst $\pi^+\pi^0$ Plot. Hier ist Peak am eindeutigsten und es lässt sich $D^0 \to \rho^+ K^- \to K^-\pi + \pi^0$ sehen.