

3 Präsenzblatt vom 01.11.12

Aufgabe 3

$$\begin{aligned}
 \text{i) } a_1 a_1 a_1^\dagger a_2 a_2^\dagger &= a_1 a_1 a_1^\dagger (\delta_{22} - a_2^\dagger a_2) = a_1 a_1 a_1^\dagger - a_1 a_1 a_1^\dagger a_2^\dagger a_2 \\
 &= a_1 (1 - a_1^\dagger a_1) - a_1 (1 - a_1^\dagger a_1) a_2^\dagger a_2 = a_1 - a_1 a_1^\dagger a_1 - a_1 a_2^\dagger a_2 + a_1 a_1^\dagger a_1 a_2^\dagger a_2 \\
 &= a_1 - (1 - a_1^\dagger a_1) a_1 - (0 - a_2^\dagger a_1) a_2 + (1 - a_1^\dagger a_1) (0 - a_2^\dagger a_1) a_2 \\
 &= a_1 - a_1 + a_1^\dagger a_1 a_1 + a_2^\dagger a_1 a_2 + a_1^\dagger a_1 a_2^\dagger a_1 a_2 - a_2^\dagger a_1 a_2 \\
 &= a_1^\dagger a_1 a_1 + a_2^\dagger a_1 a_2 - a_2^\dagger a_1 a_2 + a_1^\dagger (0 - a_2^\dagger a_1) a_1 a_2 \\
 &= a_1^\dagger a_1 a_1 - a_1^\dagger a_2^\dagger a_1 a_1 a_2 \\
 &\Rightarrow \langle 0 | a_1^\dagger a_1 a_1 - a_1^\dagger a_2^\dagger a_1 a_1 a_2 | 0 \rangle = \langle 0 | a_1^\dagger a_1 a_1 | 0 \rangle - \langle 0 | a_1^\dagger a_2^\dagger a_1 a_1 a_2 | 0 \rangle = 0
 \end{aligned}$$

$$\begin{aligned}
 \text{ii) } a_1 a_2 a_2^\dagger a_1^\dagger &= a_1 (1 - a_2^\dagger a_2) a_1^\dagger = a_1 a_1^\dagger - a_1 a_2^\dagger a_2 a_1^\dagger \\
 &= 1 - a_1^\dagger a_1 - (0 - a_2^\dagger a_1) (0 - a_1^\dagger a_2) \\
 &= 1 - a_1^\dagger a_1 - a_2^\dagger a_2 + a_2^\dagger a_1^\dagger a_1 a_2 \\
 &\Rightarrow \langle 0 | 1 - a_1^\dagger a_1 - a_2^\dagger a_2 + a_2^\dagger a_1^\dagger a_1 a_2 | 0 \rangle \\
 &= \langle 0 | 1 | 0 \rangle - \langle 0 | a_1^\dagger a_1 | 0 \rangle - \langle 0 | a_2^\dagger a_2 | 0 \rangle + \langle 0 | a_2^\dagger a_1^\dagger a_1 a_2 | 0 \rangle = 1
 \end{aligned}$$

Zusatzaufgabe

$$\begin{aligned}
 [\hat{f}, \hat{N}] &= \sum_{i,m,n} f_{m,n} [a_i^\dagger a_i, a_m^\dagger a_n] \\
 &= \sum_{i,m,n} f_{m,n} (a_i^\dagger \underbrace{[a_i, a_m^\dagger]_+}_{\delta_{mi}} a_n - \underbrace{[a_i^\dagger, a_m^\dagger]_+}_{=0} a_i a_n + a_m^\dagger a_i^\dagger \underbrace{[a_i, a_n]_+}_{=0} - a_m^\dagger \underbrace{[a_i^\dagger, a_n]_+}_{\delta_{in}} a_i) \\
 &= \sum_{i,m,n} f_{m,n} (a_i^\dagger a_n \delta_{mi} - a_m^\dagger a_i \delta_{ni}) = \sum_{m,n} f_{m,n} (a_m^\dagger a_n - a_m^\dagger a_n) = 0
 \end{aligned}$$