Theo 5 - Thermodynamik, Tutorium

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1 Tutorium vom 02.11.2011

1.1 Beispiel zur Gamma-Funktion

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$$
$$\Gamma(\tfrac{1}{2}) = \int_0^\infty t^{-\frac{1}{2}} e^{-t} = \int_0^\infty \frac{e^{-t}}{\sqrt{t}} dt$$
$$\stackrel{t:=x^2}{=} \int_0^\infty \frac{e^{-x^2}}{x} 2x dx = \sqrt{\pi}$$

Hinweis:

$$\int_{-\infty}^\infty \int_{-\infty}^\infty e^{-x^2-y^2} dxdy \stackrel{Polarkoord}{=} 2\pi \int_0^\infty r e^{-r^2} dr = 2\pi \cdot \tfrac{1}{2} [e^{-r^2}]_0^\infty = \pi$$
$$= \int_{-\infty}^\infty e^{-x^2} dx \int_{-\infty}^\infty e^{-y^2} dy = \left(\int_{-\infty}^\infty e^{-x^2} dx \right)^2 = \pi$$
$$\Rightarrow \int_{-\infty}^\infty e^{-x^2} dx = \sqrt{\pi}$$

1.2 Stirling für ln(N!)

$$\ln(N!) = \sum_{n=1}^N \ln(n) \approx \int_1^N \ln(x) dx = [x \ln(x) - x]_1^N = N \ln(N) - N + 1 \approx N \ln(N) - N$$

1.3 Binomialverteilung

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$

$\binom{n}{k} p^k (1-p)^{n-k}$: p: Trefferwahrschk bei 1 Versuch, n: Versuche, k: Treffer

Wahrscheinlichkeit bei n Versuchen k Treffer zu erreichen.

p=40%, n=10, 6 Treffer:

$P(10, 6, 40\%) = \binom{10}{6} \cdot 0.4^5 \cdot 0.6^4 = 11.15\%$

Min. 6 Treffer: $P(k \geq 6) = P(6) + P(7) + P(8) + P(9) + P(10) = 16.62\%$

84 Studenten im ersten Semester, Wahrschk. in einer Klausur zu bestehen: 70%

Man muss 2 Klausuren bestehen. Wie wahrscheinlich, dass man besteht?

$P(n = 2, k = 2, p = 0.7) = 49\%$

45 Studenten, 8 Klausuren:

Klausuren:	2	2	2	2
Wahrschk:	0.7	0.9	0.5	0.8

$45 * 0.8^2 * 0.5^2 * 0.7^2 * 0.9^2 \approx 3$

2 Tutorium vom 09.11.2011

2.1 Wiederholung: Delta-Funktion

Werden benutzt zur Transformation einer kontinuierlichen Funktion zu einer diskreten Schreibweise.

$\int\limits_{-\infty}^{\infty} f(x)\delta(x-a)dx = f(a)$

$\delta[f(x)] = \sum\limits_i \frac{1}{|f'(x_i)|}\delta(x-x_i)$, $\quad x_i$:einfache Nullstelle der Funktion

$\int\limits_{-\infty}^{x} \delta(x)dx = \theta(x)$

$\int\limits_{-\infty}^{\infty} f(x)\delta'(x) = -\int\limits_{-\infty}^{\infty} f'(x)\delta(x)dx = -f'(0)$

2.2 Delta-Funktion: Übungen

1) $\int\limits_{\alpha}^{\beta}(f(x)-f(a))\delta(x-a)dx = 0$

2) $\int\limits_{0}^{\infty} \ln(x)\delta'(x-a)dx = \begin{cases} -\frac{1}{a} & a \geq 0 \\ 0 & sonst \end{cases}$

3) $\int\limits_{0}^{\pi} \sin(\theta)\delta(\cos(\theta)-\cos(\frac{\pi}{3})d\theta = 1$

4) $\int\limits_{0}^{5} \sqrt{x}\delta(x^3-7x^2+16x-12)dx = \int\limits_{0}^{5} \sqrt{x}\frac{1}{3x^2-14x+16}\delta(x-3) = \sqrt{3}$

5) $\int\limits_{-\infty}^{\infty} x\delta(x^2-a^2)dx = 0$

6) $\int\limits_{1/2}^{5/2} e^{-x}\delta(x^3-6x^2+11x-6)dx = \frac{1}{2e}+\frac{1}{e^2}$

2.3 Blatt2, Augabe6: Liouville-Gleichung

$\rho_0(\vec{q},\vec{p})$

$\rho(\vec{q},\vec{p},t) = \int d^f q_0 \int d^f p_0 \rho_0(\vec{q}_0,\vec{p}_0)\delta(\vec{q}-\vec{Q}(\vec{p}_0,\vec{q}_9,t))\delta(\vec{p}-\vec{P}(\vec{p}_0,\vec{q}_9,t))$

$\vec{Q}$ und $\vec{P}$ lösen $\frac{\partial H}{\partial p_i} = \dot{q}_i, \quad \frac{\partial H}{\partial q_i} = -\dot{p}_i$

Z.z. : $\frac{\partial \rho}{\partial t} = \sum\limits_{i=1}^{f}(\frac{\partial H}{\partial q_i}\frac{\partial \rho}{\partial p_i} - \frac{\partial H}{\partial p_i}\frac{\partial \rho}{\partial q_i}) = \{\rho, H\}$

$\frac{\partial}{\partial t}(\delta(\vec{q}-\vec{Q}(t))\delta(\vec{p}-\vec{P}(t)))$

$= (\frac{\partial}{\partial t}\delta(\vec{q}-\vec{Q}(t)))\cdot\delta(\vec{p}-\vec{P}(t)) + (\frac{\partial}{\partial t}\delta(\vec{p}-\vec{P}(t)))\delta(\vec{q}-\vec{Q}(t))$

$= \frac{\partial\delta(\vec{q}-\vec{Q}(t))}{\partial\vec{q}-\vec{Q}(t)}\cdot\frac{\partial(\vec{q}-\vec{Q}(t))}{\partial t}\delta(\vec{p}-\vec{P}(t)) + \frac{\partial\delta(\vec{p}-\vec{P}(t))}{\partial\vec{p}-\vec{P}(t)}\cdot\frac{\partial(\vec{p}-\vec{P}(t))}{\partial t}\delta(\vec{q}-\vec{Q}(t))$

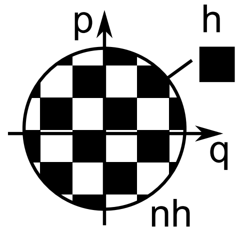
$= \frac{\partial\delta(\vec{q}-\vec{Q}(t))}{\partial\vec{Q}(t)}\cdot\frac{\partial\vec{Q}(t)}{\partial t}\delta(\vec{p}-\vec{P}(t)) + \frac{\partial\delta(\vec{p}-\vec{P}(t))}{\partial\vec{P}(t)}\cdot\frac{\partial\vec{P}(t)}{\partial t}\delta(\vec{q}-\vec{Q}(t))$

$$= \frac{\partial\delta(\vec{q}-\vec{Q}(t))}{\partial\vec{Q}(t)}\cdot\frac{\partial H}{\partial\vec{P}}\delta(\vec{p}-\vec{P}(t)) + \frac{\partial\delta(\vec{p}-\vec{P}(t))}{\partial\vec{P}(t)}\cdot\frac{\partial H}{\partial\vec{Q}}\delta(\vec{q}-\vec{Q}(t))$$

$$\int d^f q_0 \int d^f p_0 \rho_0(\vec{q}_0,\vec{p}_0)[...] = \int\int ... - \int d^f q_0 \int d^f p_0 \frac{\partial\rho_0(\vec{q}_0,\vec{p}_0)}{\partial\vec{q}}\delta(\vec{q}-\vec{Q})\delta(\vec{p}-\vec{P})\frac{\partial H}{\partial\vec{P}}$$

3 Tutorium vom 23.11.2011

3.1 Klärung: Phasenraumvolumen



$$W(E) = \frac{1}{N!}\int d^{3N}q d^{3N}p\theta(E-H)$$

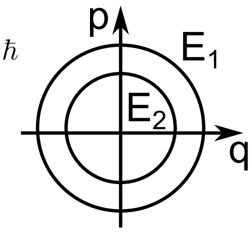
$$\Sigma(E) = \int d\Gamma\theta(E-H) = \frac{1}{N!(2\pi\hbar)^{3N}}\int d^{3N}q d^{3N}p\theta(E-H)$$

$$\Sigma(E) = \int \frac{d^{3N}pd^{3N}p}{N!(2\pi\hbar)^{3N}}\theta(E-H)$$

$$\Gamma(E) = \int d\Gamma\delta(E-H)\ , \quad \Gamma = \frac{\partial\Sigma(E)}{\partial E}$$

$W(E)$ beschreibt das Phasenraumvolumen

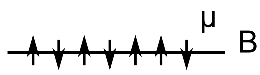
$\Sigma(E)$ beschreibt die Anzahl der Zustände im Phasenraum in Einheiten von $\hbar$



$$\Gamma(E) = \int_{E<H<E+\Delta E} d\Gamma$$

Γ beschreibt den Entartungsgrad zu einer Energie!

3.2 Aufgabe zu Zustands-Summe



n_1 Anzahl parallel in B E=-μB
n_2 Anzahl antiparallel in B E=μB

$$N >> 1, \quad E_m(n_2-n_1)\mu B(N-2n_1)\mu B$$

$$\Gamma(\tilde{E}) = \frac{N!}{n_1!n_2!} = \binom{N}{n_1} = \binom{N}{-\frac{\tilde{E}}{2\mu B}+\frac{N}{2}}$$

$$\sum_{k=0}^{N}\binom{N}{k} = 2^N = \Sigma(E)$$

$$E-(E+\delta E), \quad \mu B << dE << E$$

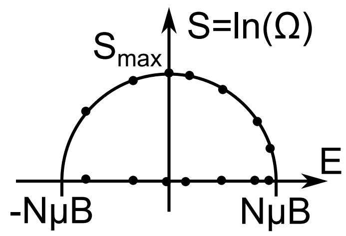
$$\Omega(E) = \sum_{E=\tilde{E}}^{E+\delta E}\Gamma(\tilde{E}) = \Gamma(\tilde{E})\sum_{\tilde{E}=E}^{E+\delta E} = \frac{\delta E}{2\mu B}\binom{N}{n_1}$$

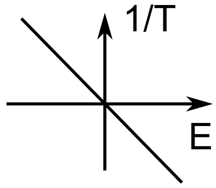
$$S = kb\ln(\Omega)$$

$$\ln(N!) = N\ln(N)-N$$

$$\ln(\Omega) = \frac{\delta E}{2\mu B}\frac{N\ln(N)-N}{(n_1\ln(n_1)-n_1)(n_2\ln(n_2)-n_2)}$$

$$\frac{1}{T} = \frac{\partial S}{\partial E} = \frac{\partial\ln(\Omega)}{\partial E}$$





4 Tutorium vom 30.11.2011

4.1 Legendre-Transformation

$$L(q_i,\dot{q}_i) \rightarrow H(q_i,p_i)\ , \quad p_i = \frac{\partial L}{\partial\dot{q}_i}$$

$$H(q_i,p_i) = \sum p_i\dot{q}_i - L(q_i,\dot{q}_i)$$

Beispiel (klassisch):

$L = \frac{1}{2}mv^2 - U \;, \quad m\dot{q}$

$H(q,p) = m\dot{q}^2 - \frac{1}{2}m\dot{q}^2 + U = \frac{1}{2}m\dot{q}^2 + U = \frac{1}{2}\frac{p^2}{m} + U$

Beispiel (Thermodynamisch):

$U(S,N,V) = -(ST - F(T,N,V)) \;, \quad S = \frac{\partial F}{\partial T}$

$U(S,N,V) = -(\frac{\partial F}{\partial T}T - F(T,N,V)) \;, \quad T \to S$

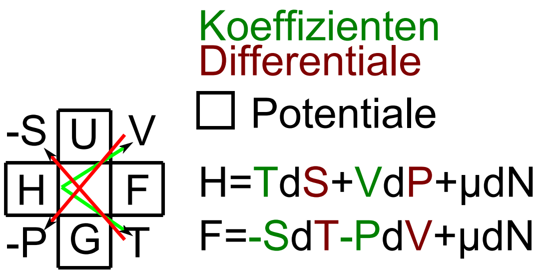
$dU = \frac{\partial U}{\partial S}\Delta S + \frac{\partial U}{\partial N}\Delta N + \frac{\partial U}{\partial V}\Delta V = Tds - pdV + \mu dN$

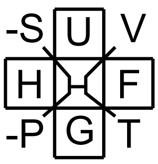
$\Rightarrow dF = SdT - pdV + \mu dN$

4.2 Gugenheim-Quadrat

Merkspruch:

Schon **u**nsere **V**orfahren **f**avorisierten **T**rinkgelage **g**egenüber **p**hysikalischen **H**erleitungen.

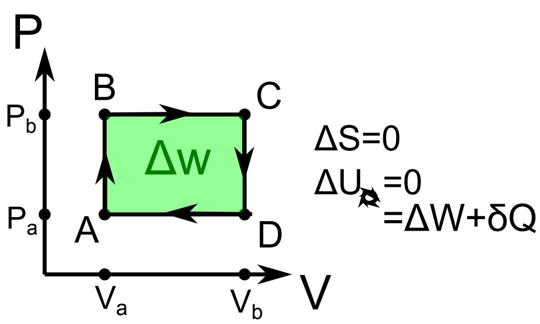




$\Rightarrow (\frac{\partial S}{\partial P})_T = (\frac{\partial V}{\partial T})_P$

5 Tutorium vom 7.12.2011

5.1 Kreisprozesse



Angabe in der Regel:

$pV = nRT, \quad U = \frac{B}{2}knT$

$A: \; V_a, T_a, p_a$

$B:\ p_b$

$C:\ V_c$

Errechnete Größen:

A	T_a	V_a	p_a
B	$T_b = \frac{p_b}{p_a} T_a$	$V_b = V_a$	p_b
C	$T_c = ...$	V_c	$p_c = p_b$
D	$T_d = ...$	$V_d = V_c$	$p_d = p_a$

(p_{13} : streng geheimer Regierungsdruck, bei dem aus Diamant Gold wird)

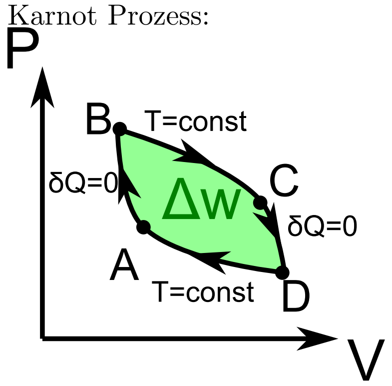
$\Delta S_{A \to B} = \int_A^B \frac{\delta Q}{T} = \int_A^B C_v \frac{dT}{T} = C_V \ln(\frac{T_b}{T_a})$

$C_v = \frac{3}{2} kN$, $C_p = \frac{5}{2} kN$

	$A \to B$	$B \to C$
ΔW	0	$p_b(V_c - V_b)$
δQ	$U(T_b) - U(T_a)$	$U(T_c) - U(T_b) - p_b(V_c - V_b)$
ΔS	$C_V \ln(\frac{T_b}{T_a})$	$C_p \ln(\frac{T_c}{T_b})$

	$C \to D$	$D \to A$
ΔW	0	$p_a(V_a - V_d)$
δQ	$U(T_d) - U(T_c)$	$U(T_d) - U(T_c) - p_a(V_a - V_d)$
ΔS	$C_V \ln(\frac{T_d}{T_c}) = -C_V \ln(\frac{T_b}{T_a})$	$C_p \ln(\frac{T_a}{T_d}) = -C_p \ln(\frac{T_c}{T_b})$

Karnot Prozess:

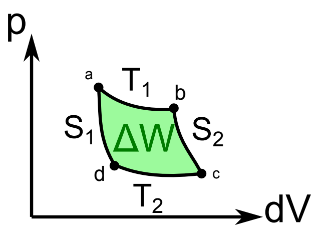


	$A \to B$	$B \to C$
ΔW	$U(T_b)_U(T_a)$	$\int_B^C p dV = \int_B^C \frac{nRT}{V} dV = nRT \ln(\frac{V_c}{V_b})$
δQ	0	$-\Delta W = T(S_1 - S_2)$

6 Tutorium vom 14.12.2011

6.1 Kreisprozesse

Carnot-Prozess



$0 = \Delta S \Leftrightarrow dQ = 0 \Leftrightarrow$ adiabatisch

$T1 > T2, \quad S2 > S1$

$pV = nkT$

Beispielangaben: $V_a, \ p_a, \ T_a, \ V_b$

$p_b = \frac{V_a}{V_b} p_a$ etc. $\Rightarrow$Damit alle Punkte ausgeben.

$\Delta U = \Delta W + \Delta Q \ , \quad \Delta W = -p(V)dV, \quad U = \frac{3}{2}kTN$

$\Rightarrow \Delta W = -\int_{V_a}^{V_b} p(v)dV - \int_{V_b}^{V_c} p(v)dV - ...$

$\oint dU = 0 = \oint \Delta W + \oint \Delta Q = 0$

$a \to b$: isotherm $\Rightarrow \ \Delta T = 0 \ \Rightarrow \ \Delta U = 0 \ \Rightarrow \ \Delta W = -\Delta Q$

$\Delta W = -\int_{V_a}^{V_b} pdV = -\int_{V_a}^{V_b} \frac{nkT_1}{V} dV = -NkT_1 \ln(\frac{V_b}{V_a}) < 0 \quad \Rightarrow \Delta Q > 0$

$b \to c$: adiabatisch $\Rightarrow \Delta U = 0 \ \Rightarrow \ \Delta U = \Delta W = \frac{3}{2}kN(T_2 - T_1)$

$c \to d$: wie $a \to b: \quad \Delta W = -NkT_2 \ln(\frac{V_d}{V_c}) > 0$

$d \to a$: wie $b \to c: \quad \Delta W = \frac{3}{2}Nk(T_1 - T_2)$

$\eta = \frac{-\Delta W}{\sum\limits_{\Delta Q > 0} \Delta Q}$

hier: $\eta = -\Delta W / \Delta Q_{a \to b}$

$\Delta U = 0 = \Delta W + \Delta Q_1 + \Delta Q_3 \quad \Rightarrow \quad -\Delta W = \Delta Q_1 + \Delta Q_3$

$\eta = \frac{\Delta Q_1 + \Delta Q_3}{\Delta Q_1} = 1 + \frac{\Delta Q_3}{\Delta Q_1}$

$\Delta Q_3 = NkT_2 \ln(\frac{V_d}{V_a}) \ , \quad \Delta Q_1 = NkT_1 \ln(\frac{V_b}{V_a})$

$T_1 V_b^{\gamma-1} = T_2 V_c^{\gamma-1} \ , \quad T_2 V_d^{\gamma-1} = T_1 V_a^{\gamma-1}$

$\Rightarrow \frac{V_c}{V_d} = \frac{V_b}{V_a}$

$\Rightarrow \eta = 1 - \frac{T_2}{T_1}$

7 Tutorium vom 11.1.2012 - Klausurbesprechung

7.1 Aufgabe 1

a) $Z = \prod_{i=1}^{N} Z_i$ (mit Z_i Zustandssumme jedes einzelnen Teilchens)

$$Z_i = \sum_{\substack{Zusände \\ \{-\epsilon,0,\epsilon\}}} e^{-\beta E_n} = e^{\beta\epsilon} + 1 + e^{-\beta\epsilon}$$

$$= 1 + 2\cosh(\tfrac{\epsilon}{kT})$$

$$\Rightarrow Z = (1 + 2\cosh(\epsilon\beta))^N$$

$$F = -\tfrac{1}{\beta}\ln(Z) = -NkT\ln(1 + 2\cosh(\tfrac{\epsilon}{kT}))$$

$$S = -\tfrac{\partial F}{\partial T} = Nk\left(\ln\left(1 + 2\cosh\left(\beta\epsilon\right)\right) - \beta\epsilon\tfrac{2\sinh(\beta\epsilon)}{1+2\cosh(\beta\epsilon)}\right)$$

b) $U = -\frac{\partial}{\partial\beta}\ln(Z) = -N\epsilon\frac{2\sinh(\frac{\epsilon}{kT})}{1+2\cosh(\frac{\epsilon}{kT})}$

$$T \to 0 \quad \Rightarrow U \to -N\epsilon$$

$$T \to \infty \quad \Rightarrow U \to 0$$

7.2 Aufgabe 2

$$p = \tfrac{NkT}{V-b} - \tfrac{d}{V^2}$$

a) $c_v = \frac{\partial U}{\partial T}|_V \;, \quad z.Z.: \; \frac{\partial c_v}{\partial V}|_T = T\frac{\partial^2 p}{\partial T^2}|_V$

$$\tfrac{\partial^2 U}{\partial V\partial T} = \tfrac{\partial^2 U}{\partial T\partial V}, \quad U = U(T,V,N) = U(S(T,V,N),V,N)$$

$$\tfrac{\partial U}{\partial V} = \tfrac{\partial U}{\partial S}\tfrac{\partial S}{\partial V} + \tfrac{\partial U}{\partial V} = T\tfrac{\partial S}{\partial V} - p = T\tfrac{\partial p}{\partial T}$$

$$\tfrac{\partial c_v}{\partial V} = \tfrac{\partial}{\partial T}(T\tfrac{\partial p}{\partial T} - p) = \tfrac{\partial p}{\partial T} + T\tfrac{\partial^2 p}{\partial T^2} - \tfrac{\partial p}{\partial T} = T\tfrac{\partial^2 p}{\partial T^2}$$

b) $\frac{\partial c_v}{\partial V} = T\frac{\partial^2 p}{\partial T^2} = T\frac{\partial}{\partial T^2}(\frac{NkT}{V-b} + \frac{a}{V^2}) = 0$

c) $dU = (\frac{\partial U}{\partial T})_{V,N}dT + (\frac{\partial U}{\partial V})_{T,V}dV = c_v(T)dT + (-p + T\frac{\partial p}{\partial T})dV$

$$= c_v(t)dT + (-\tfrac{NkT}{V-b} + \tfrac{a}{V^2} + \tfrac{NkT}{V-b})dV = c_v(T)dT + \tfrac{a}{V^2}dV$$

$$\Rightarrow U = \int c_v(T)dT - \tfrac{a}{V} + const$$

d) $dS = \frac{\partial S}{\partial U}\frac{\partial U}{\partial T}dT + dSU\frac{\partial U}{\partial V}dV + \frac{\partial S}{\partial V}dV = \frac{1}{T}\frac{\partial U}{\partial T}dT + (\frac{1}{T}\frac{\partial U}{\partial V} + \frac{p}{T})dV$

$$\tfrac{c_v(T)}{T}dT + \tfrac{1}{T}(\tfrac{\partial U}{\partial V}|_T + p)dV = \tfrac{c_v(T)}{T}dT + \tfrac{1}{T}(-p + T + \tfrac{\partial p}{\partial T} + p)$$

$$= \tfrac{c_v(T)}{T}dT + \tfrac{Nk}{V-b}dV = dS$$

$$S = \int \tfrac{c_v(T)}{T}dT + Nk\ln(V-b) + const$$

e) $dU = TdS - pdV = T\frac{\partial S}{\partial T}|_V dT + T\frac{\partial S}{\partial V}|_T dV - pdV = T\frac{\partial S}{\partial T}|_V dT + (T + \frac{\partial S}{\partial V}|_T - p)dV$

$$dw = tdx + gdy \quad \Rightarrow \quad \tfrac{\partial t}{\partial y} = \tfrac{\partial g}{\partial x} \quad \Rightarrow \quad \tfrac{\partial}{\partial V}(T\tfrac{\partial S}{\partial T}|_V) = \tfrac{\partial}{\partial T}(T\tfrac{\partial S}{\partial V}|_T - p)$$

$$\Rightarrow T\tfrac{\partial^2 S}{\partial V\partial T} = \tfrac{\partial S}{\partial V}|_T + T\tfrac{\partial^2 S}{\partial T\partial V} - \tfrac{\partial p}{\partial T} \quad \Rightarrow \quad \tfrac{\partial S}{\partial V}|_T = \tfrac{\partial p}{\partial T}|_V$$

7.3 Aufgabe 3

a) $\eta = 1 - |Q_{out}|/Q_{in}$

$\eta = -\Delta W/Q_{in} + ...$ (egal, keine Zeit, Musterlösung „ist so“)

b) $Q = Nc_p(T_i - T_f)$, $Q_{in} = Nc_p(T_C - T_B)$, $Q_{out} = Nc_p(T_A - T_I)$

$\eta = 1 - \frac{T_D - T_A}{T_C - T_B}$

$pV^{\gamma} = const$, $Tp^{\frac{1-\gamma}{\gamma}} = const$

$T_C = T_1$, $T_A = T_2$

$\Rightarrow T_D p_D^{(}\frac{1-\gamma}{\gamma}) = T_c p_c^{(}\frac{1-\gamma}{\gamma})$

$\Rightarrow \quad T_D = T_C (\frac{p_C}{p_D})^{\frac{1-\gamma}{\gamma}}$

$\Rightarrow \quad T_B = T_A (\frac{p_A}{p_B})^{\frac{1-\gamma}{\gamma}}$

$\eta = 1 - \frac{T_1 (\frac{p_1}{p_2})^{\frac{1-\gamma}{\gamma}} - T_2}{T_1 - T_2 (\frac{p_2}{p_1})^{\frac{1-\gamma}{\gamma}}} = 1 - (\frac{p_1}{p_2})^{\frac{1-\gamma}{\gamma}}$

c) $\eta_c = 1 - \frac{T_2}{T_1} = 1 - (\frac{p_1}{p_B})^{\frac{1-\gamma}{\gamma}} \geq 1 - (\frac{p_1}{p_2})^{\frac{1-\gamma}{\gamma}}$

8 Tutorium vom 25.1.2012 - Klausurbesprechung

8.1 Übung zu Quantengas

Großkanoniche Zustandssumme:

$H|p >= \epsilon |p >$

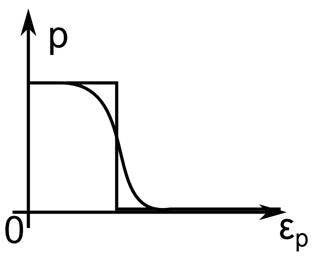
$Z = \sum\limits_{N=0}^{\infty} Sp(e^{-\beta(\hat{H} - \mu \hat{N})}) = \sum\limits_{N=0}^{\infty} \sum\limits_{\{n_p\}} e^{-\beta(\sum\limits_p n_p \epsilon_p - \mu \sum\limits_p n_p)} = \sum\limits_{n_1} \sum\limits_{n_2} ... \sum\limits_{n_p} \prod\limits_p e^{-\beta(\epsilon_p - \mu) n_p}$

$= \prod\limits_p \sum\limits_{n_p} e^{-\beta(\epsilon_p - \mu) n_p}$

Bosonengas:

$\sum\limits_{n=0}^{\infty} q^n = \frac{1}{1-q}$

$Z_B = \prod\limits_p \sum\limits_{np} \left[e^{-\beta(\epsilon_p - \mu)}\right]^{n_p} = \prod\limits_p \frac{1}{1 - e^{\beta(\epsilon_p - \mu)}}$



Fermionengas:

$n_p \in \{0, 1\}$

$\Rightarrow Z_F = \prod\limits_p \left(1 + e^{-\beta(\epsilon_p - \mu)}\right)$

$\Omega = -\frac{1}{\beta} \ln(Z)$

$\Omega_{\substack{B\\F}} = \pm \frac{1}{\beta} \sum\limits_p \ln\left(1 \mp e^{-\beta(\epsilon_p - \mu)}\right)$

$< \hat{N} >_{\substack{B\\F}} = -\frac{\partial \Omega}{\partial \mu} = \sum\limits_p \frac{1}{e^{\beta(\epsilon_p - \mu)} \mp 1} = \sum\limits_p < n_p > \quad \Rightarrow < n_p >= \frac{1}{e^{\beta(\epsilon_p - \mu)} \mp 1}$

9 Tutorium vom 1.2.2012 - Klausurvorbereitung

9.1 Stefan-Boltzmann-Gesetz über Photonengas

$TdS = dU + pdV \quad , \quad p = \frac{1}{3}\bar{u} \quad , \quad \bar{u} = \frac{U}{V} \quad , \quad \frac{\partial S}{\partial V} = \frac{\partial p}{\partial T}$

gesucht: $\bar{u} \approx f(T)$

$T\frac{\partial S}{\partial V} = \frac{\partial U}{\partial V} + p = T\frac{\partial P}{\partial T}$

$\Rightarrow \frac{1}{3}T\frac{\partial \bar{u}}{\partial T} = \bar{u} + \frac{1}{3}$

$T\frac{\partial \bar{u}}{\partial T} = 4\bar{u}$

$\frac{\partial \bar{u}}{\partial T} = \frac{4}{T}\bar{u} \quad , \quad \Rightarrow \int \frac{d\bar{u}}{\bar{u}} = 4\int \frac{dT}{T}$

$\Rightarrow \ln(\bar{u}) = 4\ln(T) = \ln(T^4) \quad \Rightarrow \quad \bar{u} = \alpha T^4$

9.2 Temperatur auf Erde als schwarzer Körper

$P = \sigma A T^4$ (σ: Boltzmann-Konstante, A: Oberfläche)

Oberfläche der Kugel mit Radius Erde-Sonne und anteilig Scheibe mit Erdradius mit Strahlungsleistung der Sonne berechnen

$W_{auf\ Erde} = \frac{\pi r^2_{Erde}}{4\pi r^2_{Sonne-Erde}} W_{von\ Sonne}$

$T = \sqrt[4]{\frac{P}{A\sigma}} = \sqrt[4]{W_{Erde}} \approx 300K$

9.3 Glühemission von Elektronen (Nolting 2.2.6)

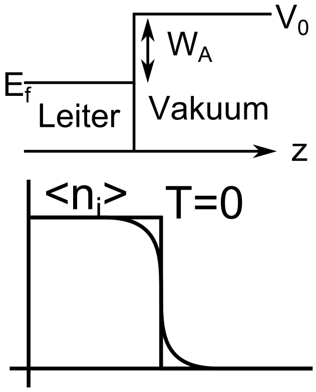
$< n_i >= \frac{1}{e^{\beta(\epsilon_i - \mu)}+1}$

$\epsilon_i = \hbar^2 k^2/2m$

$\epsilon_A = \hbar^2 k^2/2m + V_0$

$D_i = \begin{cases} d\sqrt{E} & E > 0 \\ 0 & , sonst \end{cases}$

$D_A = \begin{cases} d\sqrt{E - V_0} & E > 0 \\ 0 & , sonst \end{cases}$



$< n_i >= \frac{1}{e^{\beta(\epsilon_i - \mu)}+1}$

$< n >_i = \frac{1}{e^{\beta(\epsilon - \mu)}+1}$

$< n >_u = \frac{1}{e^{\beta(\epsilon + V_0 - \mu)}+1}$

Jetzt: Teilchenzahl außerhalb des Metalls.

$n_a = \frac{1}{V}\int_{V_0}^{\infty} dE \frac{D_a(E)}{e^{\beta(E-\mu)}+1} \quad , \quad E = \int_0^{\infty} \frac{\sqrt{E}^3}{e^{\beta(E-\mu)}+1}$

$N = \int\limits_0^\infty \frac{\sqrt{E}}{e^{\beta(E-\mu)}+1}$

$n_a = \frac{1}{V} d \int\limits_{V_0}^\infty \frac{\sqrt{E-V_0}}{e^{\beta(E-\mu)}+1}$

$\beta(V_0-\mu) >> 1 \quad \Rightarrow \quad n_A = \frac{d}{V} \int\limits_{V_0}^\infty \sqrt{E-V_0} e^{-\beta(E-\mu)} dE \quad = \frac{d}{V} \int\limits_0^\infty \sqrt{x} e^{-\beta(x+V_0-\mu)} dx$

$= \frac{d}{V b^{\frac{3}{2}}} e^{-\beta(V_0-\mu)} \underbrace{\int\limits_0^\infty \sqrt{y} e^{-y} dy}_{=\Gamma(3/2)} \quad \Rightarrow n_A = CT^{\frac{3}{2}} e^{-\beta(V_0-\mu)}$

10 Tutorium vom 8.2.2012 - Klausurvorbereitung

10.1 Glühemission

$T = 300°K \quad , \quad S = 0$

$\mu_m = \mu_v = \epsilon_F$

$n = \frac{1}{e^{\beta(\epsilon-\mu)}+1}$

$n_\sigma = 4\pi(\frac{2m}{h^2})^{\frac{3}{2}} \int\limits_0^\infty \frac{\epsilon^{\frac{1}{2}}}{e^{\beta(\epsilon+w_A-\epsilon_F)}+1} d\epsilon$

$n_m = 4\pi(\frac{2m}{h^2})^{\frac{3}{2}} \int\limits_0^\infty \frac{\epsilon^{\frac{1}{2}}}{e^{\beta(\epsilon-\epsilon_F)}+1} d\epsilon \ \beta(w_A-\epsilon_F) >> 1$

$x^2 = \epsilon$

$n_v = 4\pi(\frac{2m}{h^2})^{\frac{3}{2}} e^{-\beta(w_A-\epsilon_F)} \int\limits_0^\infty \sqrt{\epsilon} e^{-\beta\epsilon} d\epsilon$

$\int\limits_0^\infty \sqrt{\epsilon} e^{-\beta\epsilon} d\epsilon = 2 \int\limits_0^\infty x^2 e^{-\beta x^2} dx = \sqrt{\pi k^3 T^3}$

$n_v = \frac{2\pi^{\frac{3}{2}}}{h^3} (2mkT)^{\frac{3}{2}} e^{-\frac{w_A-\epsilon_F}{kT}}$

$f_m = e n_v \frac{1}{2} < |v_x| > \quad , \quad < |v_x| > = \sqrt{\frac{2kT}{\pi m}}$

$f(\mu+kTz) \approx f(\mu) + f'(\mu) z + \frac{1}{2} f''(\mu) z^2 (kT)^2$

$f(\mu-kTz) \approx f(\mu) - f'(\mu) z + \frac{1}{2} f''(\mu) z^2 (kT)^2$

$\Rightarrow I = C\big(\int\limits_0^\mu f(\epsilon) d\epsilon + 2(kT)^2 f'(\mu) \int\limits_0^\infty \frac{z}{e^z+1} dz + \frac{1}{3}(kT)^4 f'''(\mu) \int\limits_0^\infty \frac{z^3}{e^z+1} dz + ...\big)$

$I = C\big(\int\limits_0^\mu f(\epsilon) d\epsilon + \frac{\pi^2}{6}(kT)^2 f''(\mu) ...\big)$

10.2 Aufgabe zur Boseverteilung

$N = \sum\limits_i < n_i > \quad , \quad U = \sum\limits_i < n_i > \epsilon_i$

$\sum\limits_k \rightarrow \frac{V}{(2\pi)^3} \int d^3k \rightarrow 2\pi(\frac{2m}{\hbar^2})^{\frac{3}{2}} \int \epsilon^{\frac{1}{2}} d\epsilon$

$I = C \int\limits_0^\infty \frac{f(\epsilon)}{e^{\frac{\epsilon-\mu}{kT}}+1}$

$e - \mu = kTz$

$I = CkT \int\limits_{-\frac{\mu}{kT}}^\infty \frac{f(\mu+kTz)}{e^z+1} dz = kT\big(\int\limits_0^{\frac{\mu}{kT}} \frac{f(\mu-kTz)}{e^{-z}+1} dz + \int\limits_0^\infty \frac{f(\mu+kTz)}{e^2+1}\big)$

$\frac{1}{e^{-z}+1} = 1 - \frac{1}{e^{2}+1} \Rightarrow C\big(\int\limits_0^\mu f(\epsilon) + kT\int\limits_0^\infty \frac{f(\mu+kTz)-f(\mu-kTz)}{e^z+1}dz\big)$

$\Rightarrow N = \frac{2}{3}\frac{V}{(2\pi)^3}C\mu^{\frac{3}{2}}(1+\frac{\pi^2}{8}(\frac{kT}{\mu})^2 + ...)$

$U = \frac{3}{5}N\epsilon(1+\frac{5}{12}\pi^2(\frac{kT}{\epsilon_f})^2 + ...)$

$C_v = const * T + -T^3 ... \xrightarrow{T\to 0} 0$