

6.5 + 2

Nr. 1

a) Gram-Schmidt:

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 3 \\ 1 & 4 \\ 1 & 7 \end{pmatrix} \Rightarrow \text{Rang}(A) = 2$$

\Rightarrow eind. lösbar

$$b = \begin{pmatrix} 1 \\ 2 \\ 6 \\ 4 \end{pmatrix}$$

$$q_1 = \frac{a_1}{\|a_1\|_2} = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)^T$$

$$\tilde{q}_2 = a_2 - (a_2^T q_1) q_1 = \begin{pmatrix} 0 \\ 3 \\ 4 \\ 7 \end{pmatrix} - \left(\frac{3}{2} + \frac{4}{2} + \frac{7}{2}\right) \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix} = \begin{pmatrix} -7/2 \\ -1/2 \\ 1/2 \\ 7/2 \end{pmatrix}$$

$$q_2 = \frac{\tilde{q}_2}{\|\tilde{q}_2\|_2} = \left(-\frac{7}{10}, -\frac{1}{10}, \frac{1}{10}, \frac{7}{10}\right)^T$$

$$\Rightarrow Q = \begin{pmatrix} 1/2 & -7/10 \\ 1/2 & -1/10 \\ 1/2 & 1/10 \\ 1/2 & 7/10 \end{pmatrix} \quad R_{jk} = \begin{cases} \|q_j\|_2 & j=k \\ (a_k, q_j) & j < k \\ 0 & j > k \end{cases} \Rightarrow R = \begin{pmatrix} 2 & 7 \\ 0 & 5 \end{pmatrix}$$

$$R x = Q^T b \Rightarrow Q^T b = \begin{pmatrix} 1/2 & 1/2 & 1/2 & 1/2 \\ -7/10 & -1/10 & 1/10 & 7/10 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 6 \\ 4 \end{pmatrix} = \begin{pmatrix} 13/2 \\ 5/2 \end{pmatrix}$$

$$\Rightarrow \left(\begin{array}{cc|c} 2 & 7 & 13/2 \\ 0 & 5 & 5/2 \end{array} \right) \Leftrightarrow \left(\begin{array}{cc|c} 2 & 7 & 13/2 \\ 0 & 1 & 1/2 \end{array} \right) \Leftrightarrow \left(\begin{array}{cc|c} 1 & 0 & 3/2 \\ 0 & 1 & 1/2 \end{array} \right)$$

$$\Rightarrow x = \left(\frac{3}{2}, \frac{1}{2}\right)$$

b) Householder:

$\alpha = -0.5$

$$u_1 = \frac{a_1 - \alpha_1 e_1}{\|a_1 - \alpha_1 e_1\|_2} = \begin{pmatrix} 3 \\ 1 \\ 1 \\ 1 \end{pmatrix} \cdot \frac{1}{\sqrt{12}}, \quad H_1 = \mathbb{1} - 2u_1 u_1^T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 3/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & 7/6 & 1/6 & 1/6 \\ 1/2 & 1/6 & 5/6 & 1/6 \\ 1/2 & 1/6 & 1/6 & 5/6 \end{pmatrix}$$

$$H_1 A = \begin{pmatrix} -2 & -7 \\ 0 & 2/3 \\ 0 & 5/3 \\ 0 & 14/3 \end{pmatrix} \Rightarrow \tilde{A}_1 = \begin{pmatrix} 2/3 \\ 5/3 \\ 14/3 \end{pmatrix} \Rightarrow H_1 = \begin{pmatrix} -2/3 & -1/3 & -1/3 & -1/3 \\ -1/3 & 5/6 & -1/6 & -1/6 \\ -1/3 & -1/6 & 5/6 & -1/6 \\ -1/3 & -1/6 & -1/6 & 5/6 \end{pmatrix}$$

$$u_2 = \frac{\tilde{a}_1 - \alpha_2 e_1}{\|\tilde{a}_1 - \alpha_2 e_1\|_2} = \left(\frac{17}{3}, \frac{5}{3}, \frac{14}{3}\right)^T \cdot \frac{1}{\sqrt{170}} = \left(\sqrt{\frac{17}{30}}, \sqrt{\frac{5}{102}}, \sqrt{\frac{98}{255}}\right)^T$$

$$\tilde{H}_2 = \mathbb{1} - 2u_2 u_2^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 34/30 & 7/3 & 14/15 \\ 7/3 & 5/51 & 14/51 \\ 14/15 & 14/51 & 196/255 \end{pmatrix} = \begin{pmatrix} -2/5 & -1/3 & -14/5 \\ -1/3 & 46/51 & -14/51 \\ -14/5 & -14/51 & 59/255 \end{pmatrix}$$

$$\Rightarrow H_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \tilde{H}_2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$R = H_2 H_1 A = \begin{pmatrix} -2 & -7 \\ 0 & -5 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad Q^T = H_2^T H_1 = \begin{pmatrix} -2/3 & -1/3 & -1/3 & -1/3 \\ 3/10 & 1/10 & -1/10 & 3/10 \\ -5/34 & -1/34 & 29/34 & -1/34 \\ 83/170 & 11/170 & -19/170 & 67/170 \end{pmatrix}$$

$$\Rightarrow R_1 x = Q^T b_1 \Rightarrow \left(\begin{array}{cc|c} -2 & -7 & -23/2 \\ 0 & -5 & -5/2 \end{array} \right) \Leftrightarrow \left(\begin{array}{cc|c} 1 & 0 & 3/2 \\ 0 & 1 & 1/2 \end{array} \right)$$

$$\Rightarrow x = \left(\frac{3}{2}, \frac{1}{2}\right)$$

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Nr. 2

Daten: $\begin{matrix} -2 & -1 & 0 & 1 & 2 \\ -0,3 & 0,2 & 0,6 & 0,8 & 3,2 \end{matrix}$, $u(x) = c_1 + c_2 x + c_3 x^2$ 8

$$\Rightarrow A = \begin{pmatrix} 1 & -2 & 4 \\ 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix} \quad A^T A = \begin{pmatrix} 5 & 0 & 10 \\ 0 & 10 & 0 \\ 10 & 0 & 34 \end{pmatrix}$$

$$b = (-0,3; 0,2; 0,6; 0,8; 3,2)^T \quad A^T b = \begin{pmatrix} 4,5 \\ 7,6 \\ 12,6 \end{pmatrix} \quad \checkmark$$

$$A^T A c = A^T b :$$

$$\left(\begin{array}{ccc|c} 5 & 0 & 10 & 4,5 \\ 0 & 10 & 0 & 7,6 \\ 10 & 0 & 34 & 12,6 \end{array} \right) \Leftrightarrow \left(\begin{array}{ccc|c} 5 & 0 & 10 & 4,5 \\ 0 & 1 & 0 & 0,76 \\ 0 & 0 & 14 & 3,6 \end{array} \right) \Leftrightarrow \left(\begin{array}{ccc|c} 5 & 0 & 10 & 4,5 \\ 0 & 1 & 0 & 0,76 \\ 0 & 0 & 1 & 9/35 \end{array} \right)$$

$$\Leftrightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 27/70 \\ 0 & 1 & 0 & 19/25 \\ 0 & 0 & 1 & 9/35 \end{array} \right) \Rightarrow u(x) = \frac{27}{70} + \frac{19}{25}x + \frac{9}{35}x^2 \quad \checkmark$$

$$r = Ac - b = \left(-\frac{37}{350}, -\frac{41}{350}, \frac{27}{70}, \frac{491}{350}, \frac{1027}{350} \right)^T - \begin{pmatrix} -0,3 \\ 0,2 \\ 0,6 \\ 0,8 \\ 3,2 \end{pmatrix}$$

$$\stackrel{\textcircled{1}}{=} \left(\frac{34}{175}, -\frac{711}{350}, -\frac{3}{70}, \frac{211}{350}, -\frac{93}{350} \right)$$

$$\|r\|_2 = \sqrt{\frac{547}{875}} \approx 0,786 \quad \checkmark \quad \frac{211}{350} \quad -\frac{93}{350} \quad \textcircled{-1}$$