

Besprechung in Mathematik für Physiker II zur 1. Klausur

Nr 2

c) $\overline{A \cup B} = \overline{A} \cup \overline{B}$

Bew: „ $\subset$ “: Sei $x \in \overline{A \cup B}$

Es ex. $(x_n) \subset A \cup B, x_n \rightarrow x$

Es ex. Teilf. $(x_{\varphi(n)}) \subset (x_n), (x_{\varphi(n)}) \subset A$ oder $(x_{\varphi(n)}) \subset (x_n), (x_{\varphi(n)}) \subset B$.

Im ersten Fall: $x_{\varphi(n)} \rightarrow x, (x_{\varphi(n)}) \subset A$. Also $x \in \overline{A}$ zweiten Fall analog: $x \in \overline{B}$.

Also $x \in \overline{A} \cup \overline{B}$

„ $\supset$ “: Sei $x \in \overline{A} \cup \overline{B}$. Falls $x \in \overline{A}$, so ex, Folge $(x_n) \subset A, x_n \rightarrow x$.

Dann auch $(x_n) \subset A \cup B$, also $x \in \overline{A \cup B}$. Analog $B \subset \overline{A \cup B}$

d) $\partial(A \cup B) = \overline{A \cup B} \setminus (A \cup B)^\circ = (\overline{A} \cup \overline{B}) \setminus (A \cup B)^\circ = [\overline{A} \setminus (A \cup B)^\circ] \cup [\overline{B} \setminus (A \cup B)^\circ]$

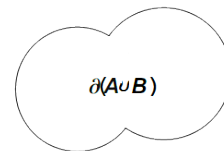
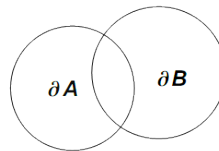
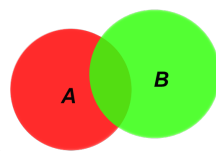
$\partial A \cup \partial B = [\overline{A} \setminus A^\circ] \cup [\overline{B} \setminus B^\circ]$

Wegen $A^\circ \subset (A \cup B)^\circ, B^\circ \subset (A \cup B)^\circ \Rightarrow \partial A \cup \partial B \supset \partial(A \cup B)$

b) x_0 isol. P

Also ex. $\delta > 0 : B_{||\cdot||}(x_0, \delta) \cap M = \{x_0\}$.

Sei $\varepsilon > 0$. Für $x \in M, ||x - x_0|| < \delta$ gilt $x = x_0, ||f(x) - f(x_0)|| = ||0|| < \varepsilon$



Nr 5

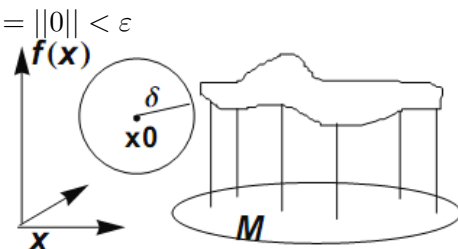
b) $A = \begin{pmatrix} 1 & 3 & 0 \\ 2 & -1 & 3 \\ -2 & 4 & 1 \end{pmatrix}. \hat{A} = (\mathbb{R}^3, ||\cdot||_2) \rightarrow (\mathbb{R}^3, ||\cdot||_\infty)$

$||A|| = \sup_{||x||_2 \leq 1} ||Ax||_\infty$

Falls $x = (x_1, x_2, x_3), ||x||_2 \leq 1 \Rightarrow |x_i| < 1, i = 1, 2, 3$.

Also $||Ax||_\infty = \max\{|x_1 + 3x_2|, |2x_1 - x_2 + 3x_3|, |-2x_1 + 4x_2 + x_3|\} \leq \max\{1 + 3, 2 + 1 + 3, 2 + 4 + 1\} = 7$

c) $||Ax||_\infty = \max\{ \underbrace{|x_1 + 3x_2|}_{\leq ||(1,3,0)||_2 - ||x||_2 \leq 1}, \underbrace{|2x_1 - x_2 + 3x_3|}_{\leq ||(2,1,3)||_2 - ||x||_2 \leq 1}, \underbrace{|-2x_1 + 4x_2 + x_3|}_{\leq ||(-2,4,1)||_2 - ||x||_2 \leq \sqrt{21}} \} \leq \sqrt{21}$



Nr 3

a) $J_f(r\varphi) = \begin{pmatrix} 1 + \cos(\varphi) & -r \sin(\varphi) \\ \sin(\varphi) & r \cos(\varphi) \\ 2r & 0 \end{pmatrix}$

$J_g(x, y, z) = \begin{pmatrix} y, x, 0 \\ 1, 0, -1 \end{pmatrix}$

$J_{g \circ f}(r, \varphi) = J_g(f(r, \varphi)) \cdot J_f(r, \varphi) = \begin{pmatrix} rs & r + rc & 0 \\ 1 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 + c & -rs \\ s & rc \\ 2r & 0 \end{pmatrix}$

$= \begin{pmatrix} rs(1 + c) + r(1 + c)s & -rs^2 + r^2c + rc^2 \\ 1 + c - 2r & -rc \end{pmatrix} = \begin{pmatrix} 2rs(1 + c) & r(c^2 - s^2) + r^2c \\ 1 + c - 2r & -rs \end{pmatrix}$

mit $s = \sin(\varphi), c = \cos \varphi$

Test: $(g \circ f)(r, \varphi) = (r(1 + \cos(\varphi))r \sin(\varphi), r(1 + \cos(\varphi) - r^2) = (r^2(1 + \cos(\varphi) \sin(\varphi), r(1 + \cos(\varphi) - r^2)$

$J_{g \circ f} = \begin{pmatrix} 2r(1 + c)s & r^2(c^2 - s^2) + r^2c \\ -2r + 1 + c & -rs \end{pmatrix}$